

Near field speckle correlations

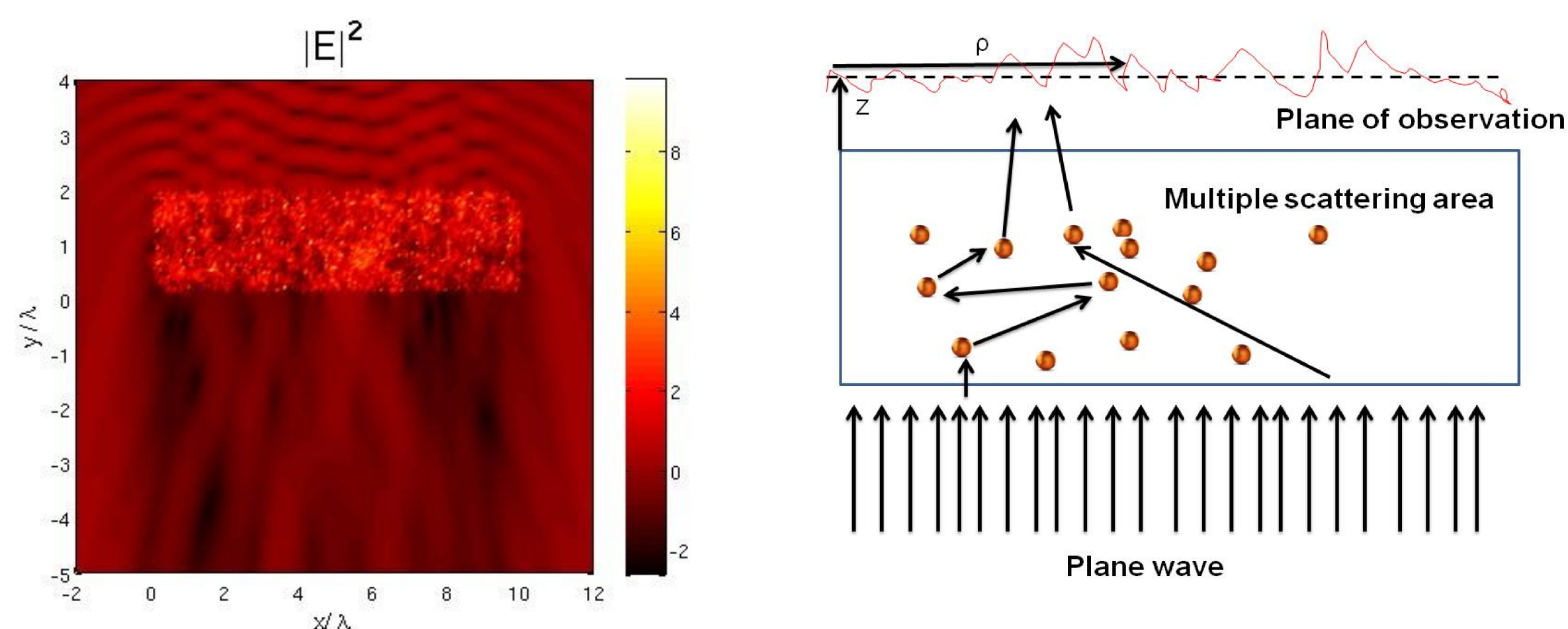
Mohamed EL Abed

Lycée Paul Eluard, Saint-Denis. mohamed_elabed@hotmail.com

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Introduction: The speckle [1]

Wave scattering by a disordered medium generates a complex intensity distribution called speckle. If the statistical properties of speckle patterns in the far field have been extensively studied, those of speckle patterns measured near field of the output surface of the medium are less well known (here we mean a near-field measured at a distance far sub-wavelength). In this area, non-propagative fields (evanescent waves) are supposed to dominate and influence largely on the statistical properties of speckle patterns.



Theoretical tools and numerical model [2,3]

1. Model of a correlated disorder

We consider a medium described by a dielectric constant of the form:

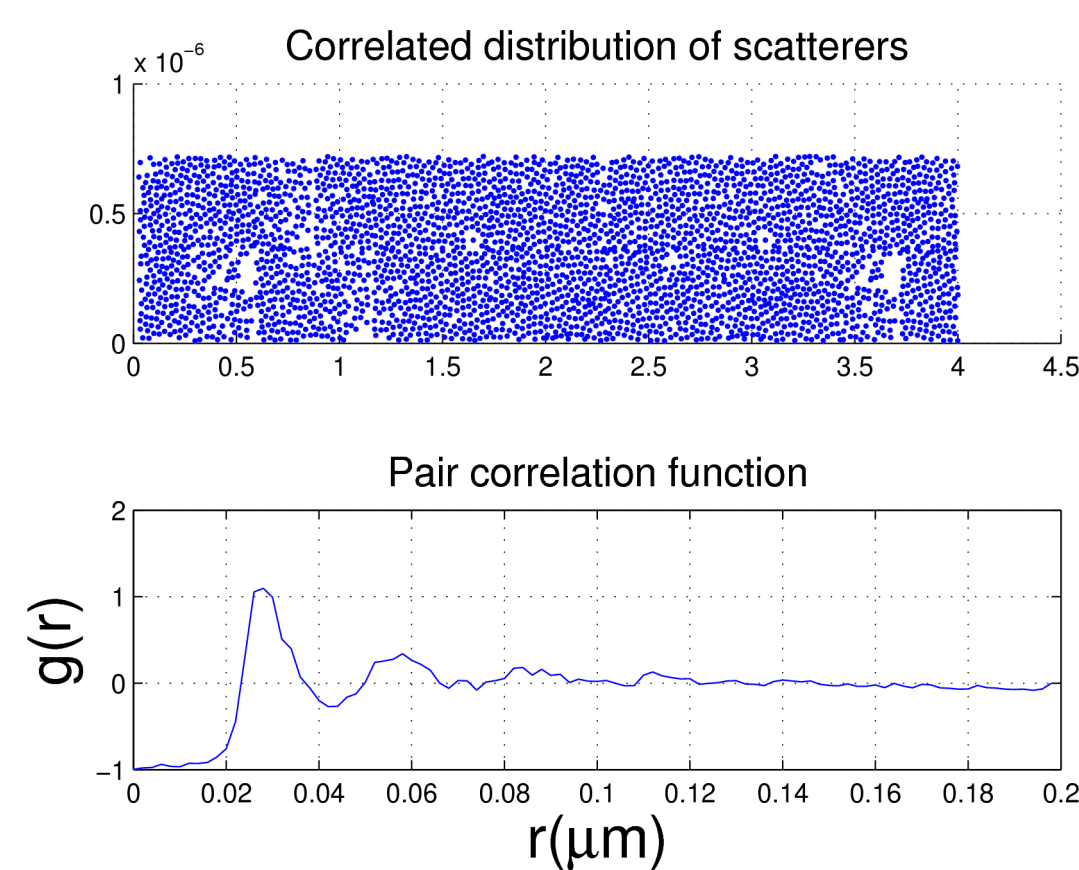
$$\epsilon(\vec{r}) = 1 + \delta\epsilon(\vec{r})$$

where the fluctuation $\delta\epsilon(\vec{r})$ satisfies: $\langle \delta\epsilon(\vec{r}) \rangle = 0$

$$\text{and } \langle \delta\epsilon(\vec{r}) \delta\epsilon(\vec{r}') \rangle = \frac{8}{\pi l_s l_c^2 k_0^3} e^{-\frac{|\vec{r}-\vec{r}'|}{l_c}}$$

l_s is the scattering mean free path and l_c is the correlation length of the medium.

The average is performed over the ensemble realizations of random distributions of the scatterers.



2. Electric field

$E(\vec{r})$ obeys the vector propagation equation:

$$\nabla \times \nabla \times E(\vec{r}) - \epsilon(\vec{r}) k_0^2 E(\vec{r}) = \vec{0} \quad (1)$$

with $E(\vec{r}) = \langle E(\vec{r}) \rangle + \langle \delta E(\vec{r}) \rangle$

$\langle E(\vec{r}) \rangle$ is the averaged electric field solution of (1) in the effective medium ($\epsilon(\vec{r}) = \epsilon_{eff}$)

$$\delta E(\vec{r}) = k_0^2 \int \langle G(\vec{r}, \vec{r}') \rangle \delta\epsilon(\vec{r}') E(\vec{r}') d\vec{r}'$$

$\langle G(\vec{r}, \vec{r}') \rangle$ is the averaged Green tensor which gives the electric field at a point \vec{r} located outside the scattering medium due to the scatterer located at the point \vec{r}' inside the medium.

3. Total Spatial correlation function

The purpose is to calculate the total correlation function:

$$\gamma_E(\vec{r}, \vec{r}') = \sum_k \langle \delta E_k(\vec{r}) \delta E_k^*(\vec{r}') \rangle$$

with:

$$\langle \delta E_k(\vec{r}) \delta E_k^*(\vec{r}') \rangle = \mu_0^2 \omega^2 \int \langle G_{km}(\vec{r}, \vec{r}_1) \rangle \langle G_{ln}^*(\vec{r}', \vec{r}'_1) \rangle \langle j_m(\vec{r}_1) j_n^*(\vec{r}'_1) \rangle d\vec{r}_1 d\vec{r}'_1$$

and:

$$\langle j_m(\vec{r}_1) j_n^*(\vec{r}'_1) \rangle = \epsilon_0^2 \omega^2 \langle \delta\epsilon(\vec{r}_1) \delta\epsilon(\vec{r}'_1) \rangle \langle E_m(\vec{r}_1) E_n^*(\vec{r}'_1) \rangle \delta_{lm}$$

4. Numerical calculations using the coupled dipoles method

Let Σ be a surface enclosing N scattering dipoles each of polarisability $\alpha(\omega)$

The total polarization of the medium is given by:

$$\vec{P}(\vec{r}') = \sum_{j=1}^N \vec{p}(\vec{r}') \delta(\vec{r}' - \vec{r}_j) \text{ with } \vec{p}(\vec{r}') = \epsilon_0 \alpha(\omega) \vec{E}_{inc}(\vec{r}')$$

Assumptions:

1. The dipole distribution is homogeneous and isotropic
2. The Born approximation is satisfied (simple diffusion)
3. The incident wave is a planewave
4. The size of the dipoles to be much smaller than λ

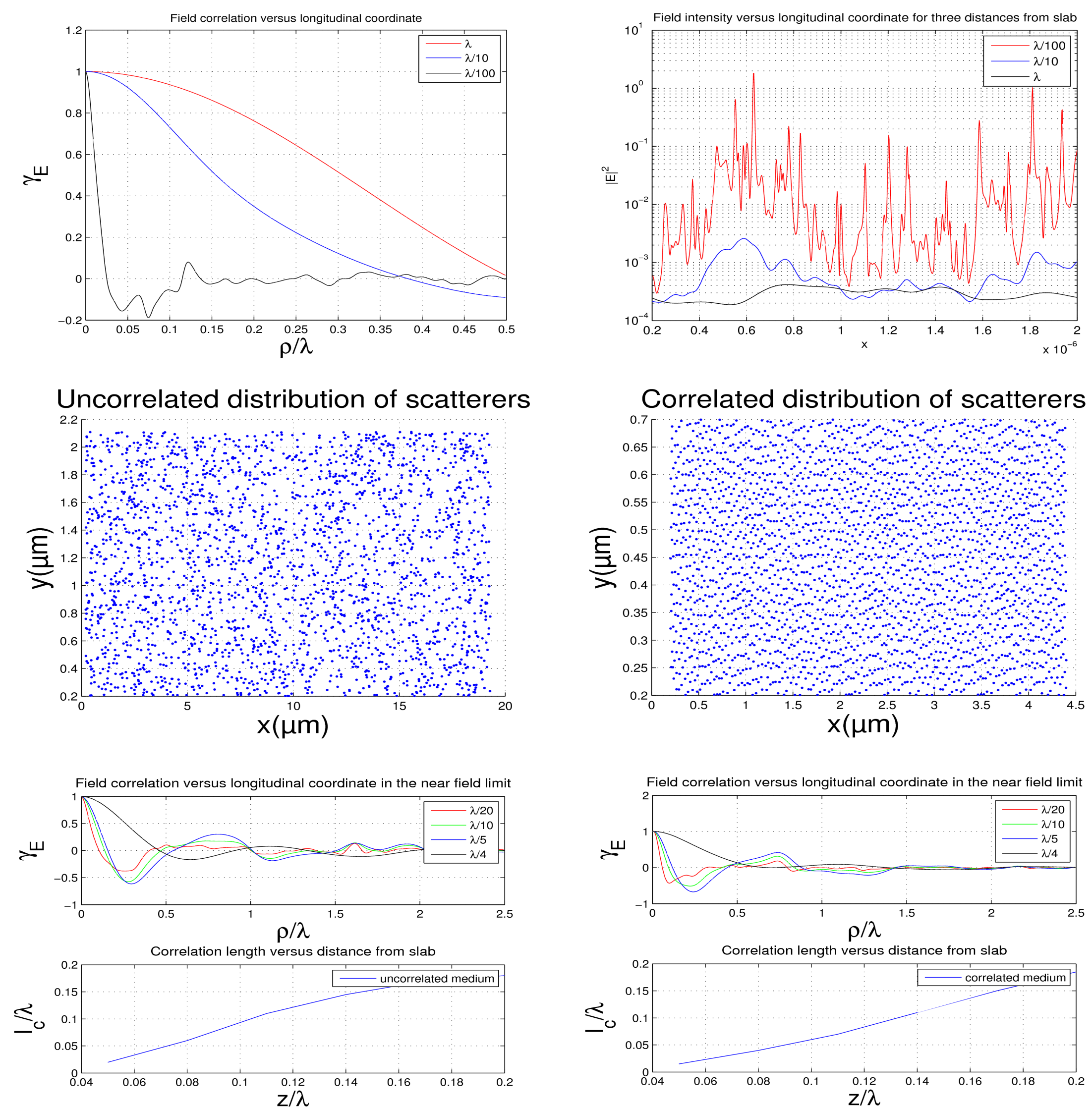
The electric field is then given by:

$$\vec{E}(\vec{r}) = \vec{E}_{inc}(\vec{r}) + \mu_0 \omega^2 \int \mathbf{G}_0(\vec{r}, \vec{r}', \omega) \vec{P}(\vec{r}') d\vec{r}'$$

$$\vec{E}(\vec{r}) = \vec{E}_{inc}(\vec{r}) + \mu_0 \epsilon_0 \omega^2 \alpha(\omega) E_0 \sum_{j=1}^N \mathbf{G}_0(\vec{r}, \vec{r}_j) \vec{E}_{inc}(\vec{r}_j) \text{ with } \vec{E}_{inc}(\vec{r}_j) = \vec{E}_0 e^{i(\vec{k}_{inc} \cdot \vec{r}_j)}$$

We can therefore calculate the electric field at every points \vec{r}_j by means of a simple matrix inversion.

Results of simulations



Conclusion and perspectives

- Non-universal statistical properties
- Strong influence of the micro structure of the samples.
- The study of near-field speckle is justified by the existence of imaging techniques based on measuring the spatial correlations of field or intensity, and the possibility of focusing waves through scattering media by time reversal or control of the wavefront.
- The lower limit of the size of a grain of speckle, that influences the spatial resolution, is not limited by diffraction in the near-field regime.

References

1. A. Ishimaru, *Wave propagation in random media*, academic, New York, 1978
2. R. Carminati, *Phys. Rev. A* 81, 053804 (2010)
3. F. Bordas et al, *Phys. Rev. E* 73, 056601 (2006)