# Localized 4D Elastic Full Waveform Inversion

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## Abstract

Full waveform inversion (FWI) is a powerful tool to quantify elastic earth propoerties of the subsurface from seismic data. The idea is based on minimizing the difference between observed data and synthetic data using adjoint-state technique. However, because of very high computational cost, FWI has so far been used mainly for 2D elastic, 3D acoustic media. The extension to 3D elastic case with realistic model size is still challenging even on current computer architecture. Here we propose an efficient way to perform localized 3D elastic FWI using grid injection method (GIM) and exact wavefield extrapolation (WE). This localized FWI is well suited for time-lapse seismic studies since it allows for efficient calculation of synthetic seismograms after model alterations within a localized area, which will lead to significant reductions in computational cost and memory requirements.

## Motivation

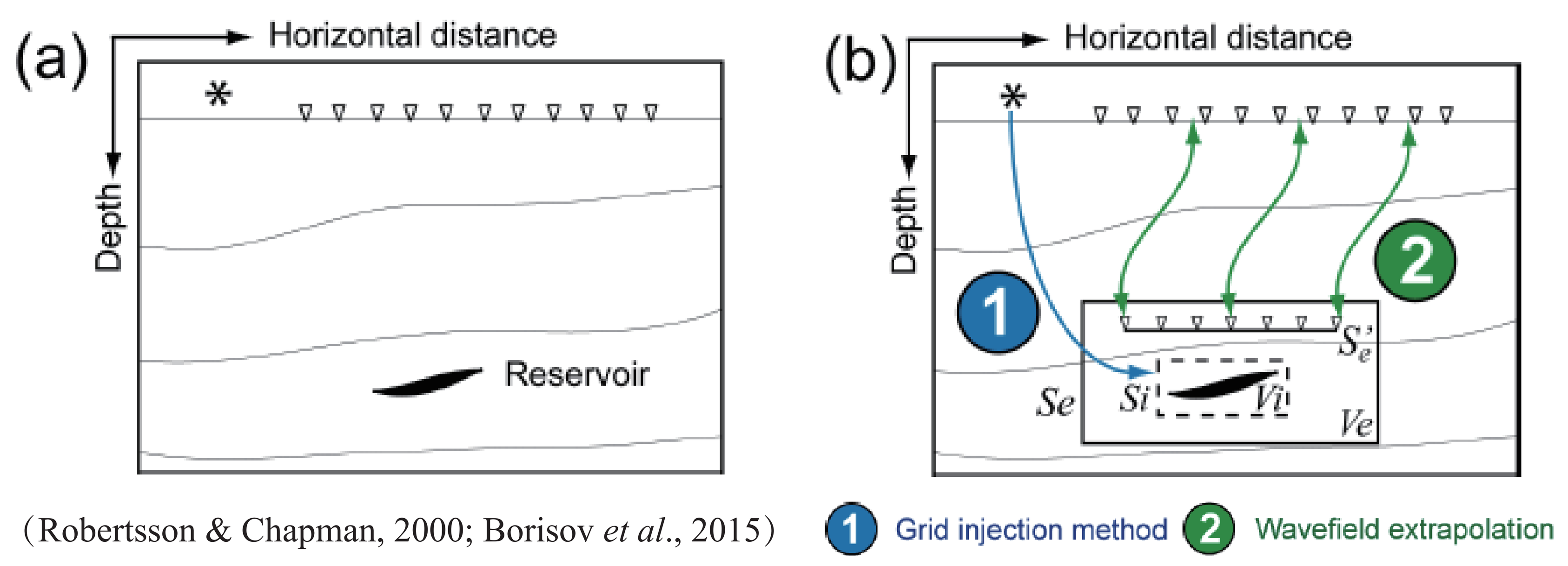



*Figure 1. Schematic diagram showing the basic concept of GIM and WE. Initial model with source & receivers placed at the surface (top) and new source & receivers position close to the reservoir (bottom)*

- Grid Injection Method (GIM) allows the recalculation of seismic response over a specific localized model through insertion of the wavefield inside a finite difference grid, without recalculating the wavefield for the whole model space;
- The wavefield recorded along a closed surface can be used as a source wavefield to compute the wavefield within the target region;
- Well suited for time lapse seismic processing (monitoring oil/gas reservoir or $CO_2$ storage);
- Seismograms are updated only in the area around model alteration.

## Grid Injection Method

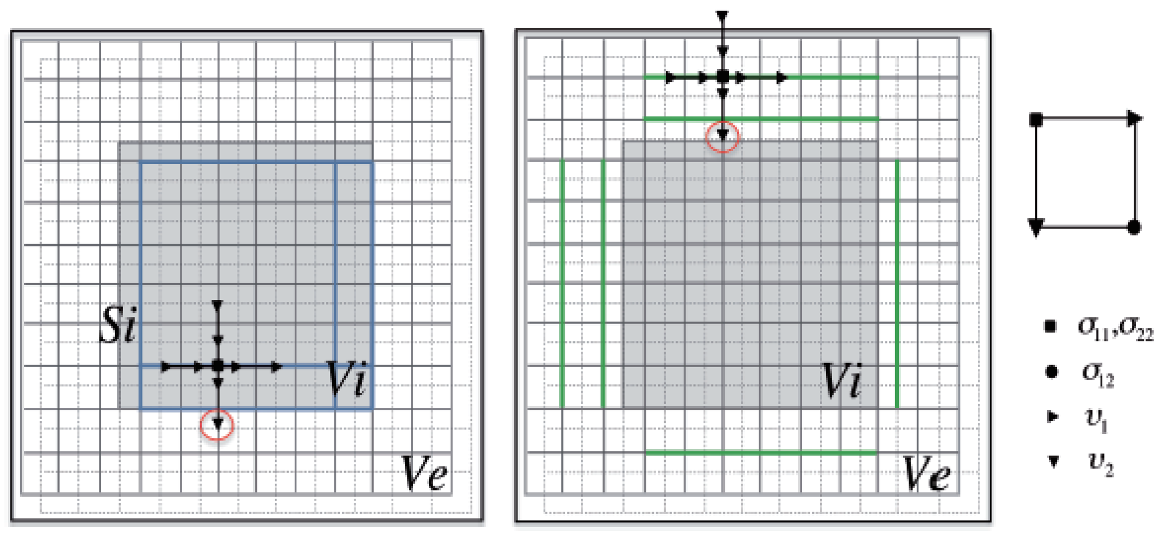
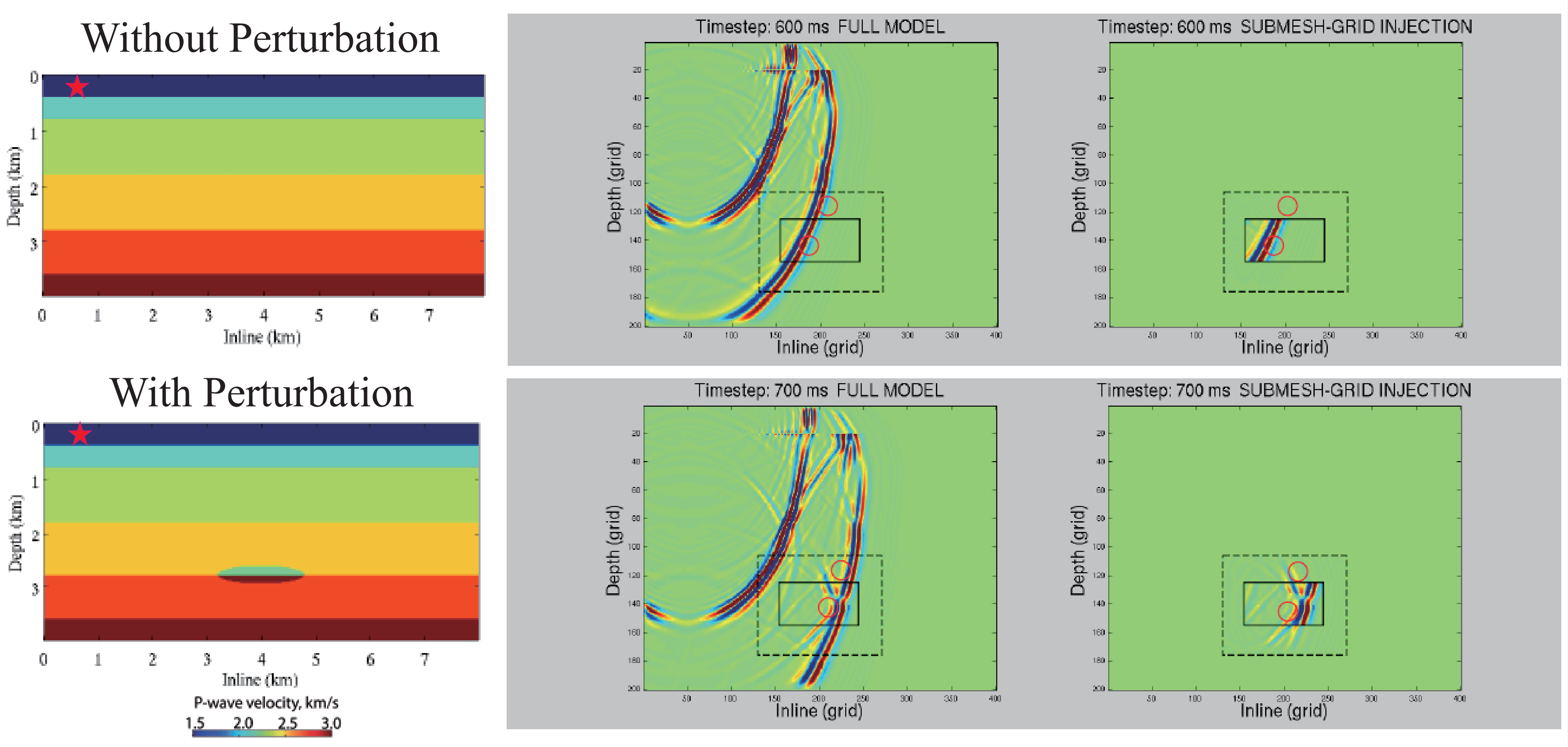


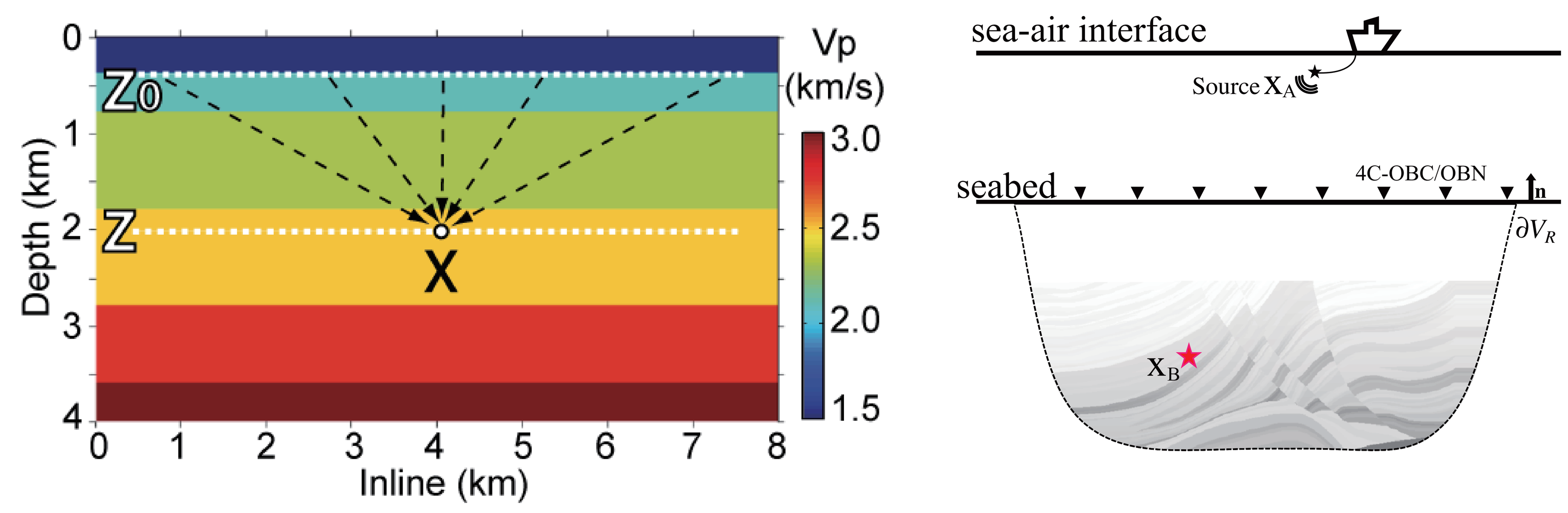

*Figure 2. Schemetic illustration of updating wavefield in staggered grids during wavefield injection.*

**Wavefield Updating**

- Inside $V_i$ : Boundary wavefield should be added.
- Outside $V_i$ : Boundary wavefield should be subtracted.




## Wavefield Extrapolation




Correlation-type wavefield representation (Wapenaar *et al.*, 2006; Ravasi *et al.*, 2014):

$$\hat{D}^{v,f(S)^*}_{q,p}(\mathbf{x}_B,\mathbf{x}_A) = -\oint_{\partial \mathbb{D}_0} \left(\hat{D}^{\tau,f(S)^*}_{ij,p}(\mathbf{x},\mathbf{x}_A)\hat{G}^{v,f(0)}_{i,q}(\mathbf{x},\mathbf{x}_B) + \hat{D}^{v,f(S)^*}_{i,p}(\mathbf{x},\mathbf{x}_A)\hat{G}^{\tau,f(0)}_{ij,q}(\mathbf{x},\mathbf{x}_B)\right) n_j \, d^2\mathbf{x}$$

In ocean bottom seismics: $\mathbf{v}(\mathbf{x}_R,t) = \begin{pmatrix} v_x(\mathbf{x}_R,t) \\ v_y(\mathbf{x}_R,t) \\ v_z(\mathbf{x}_R,t) \end{pmatrix}$, $\mathbf{t}_z(\mathbf{x}_R,t) = \begin{pmatrix} \tau_{xz}(\mathbf{x}_R,t) \\ \tau_{yz}(\mathbf{x}_R,t) \\ \tau_{zz}(\mathbf{x}_R,t) \end{pmatrix} \overset{\text{fluid-solid boundary}}{=} \begin{pmatrix} 0 \\ 0 \\ -p(\mathbf{x}_R,t) \end{pmatrix}$

## Wavefield Extrapolation Examples

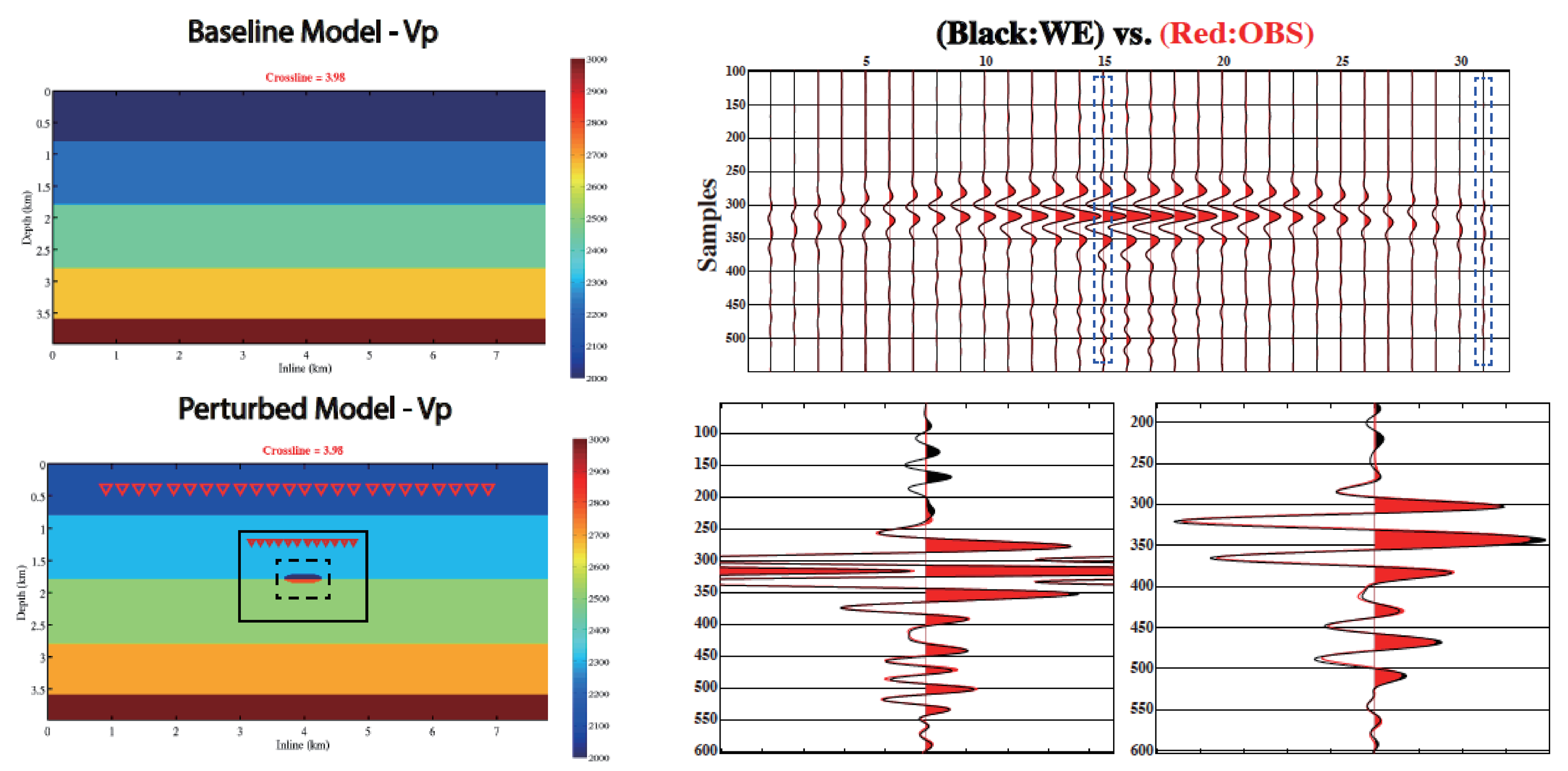



*Figure 3. An example of wavefield extrapolation. As is shown above, the extrapolated wavefield (black curves) is quite consistent with the true wavefield at the corresponding positions.*

## Localized FWI

### Forward Modeling

- Time-domain 1st order velocity-stress system in isotropic medium:

$$\rho \partial_t \upsilon = \nabla \cdot c, \qquad \partial_t \sigma = c : \upsilon,$$

where ρ is density, υ is velocity, σ is stress tensor and c is elastic tensor.

- The FD-scheme is based on staggered grid, 2nd order in time and 4th order in space O(Δt2, h4) (Levander, 1988).

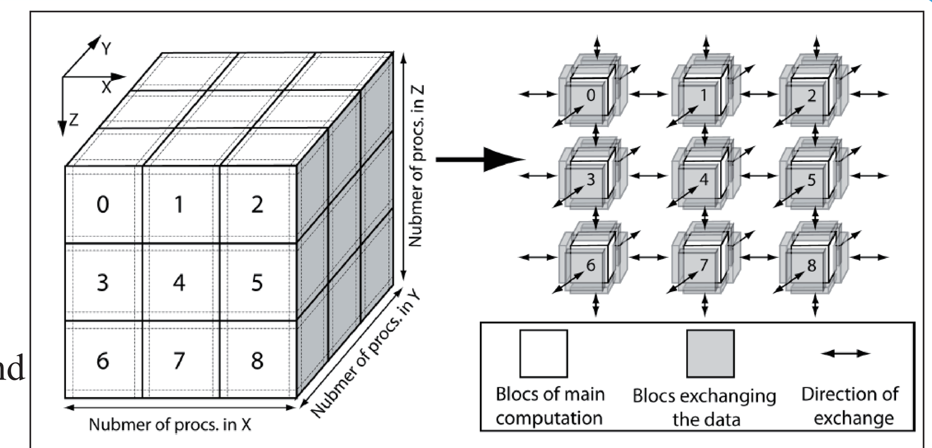



*Figure 4. Schematic illustration of spatial 3D domain decomposition. Each processor performs calculation within small blocs separately and interchanges with neighborhood processors.*

- Convolutional Perfectly Matched Layers (C-PML) were used for efficient wave absorbtion at the model boundaries (Komatitsch and Martin, 2007)
- Parallel computation (MPI): Domain-splitting and Multi-shots (figure 4).

### Inverse Problems

- Iteratively minimise the misfit $S$ between modeled data - dMOD and observed data - dOBS:

$$S = \Sigma_{shots} \int_0^T \Sigma_{recv} [d_{mod}(t) - d_{obs}(t)]^2 dt$$

- Gradient is calculated as cross-correlation between forward modeled wavefield (u) and back propagated residuals (ψ):

$$g = \sum_{shots} \int_0^T dt (\vec{u} \cdot \vec{\psi}),$$

- Density is not inverted and is updated using empirical relationship with P-wave velocity.

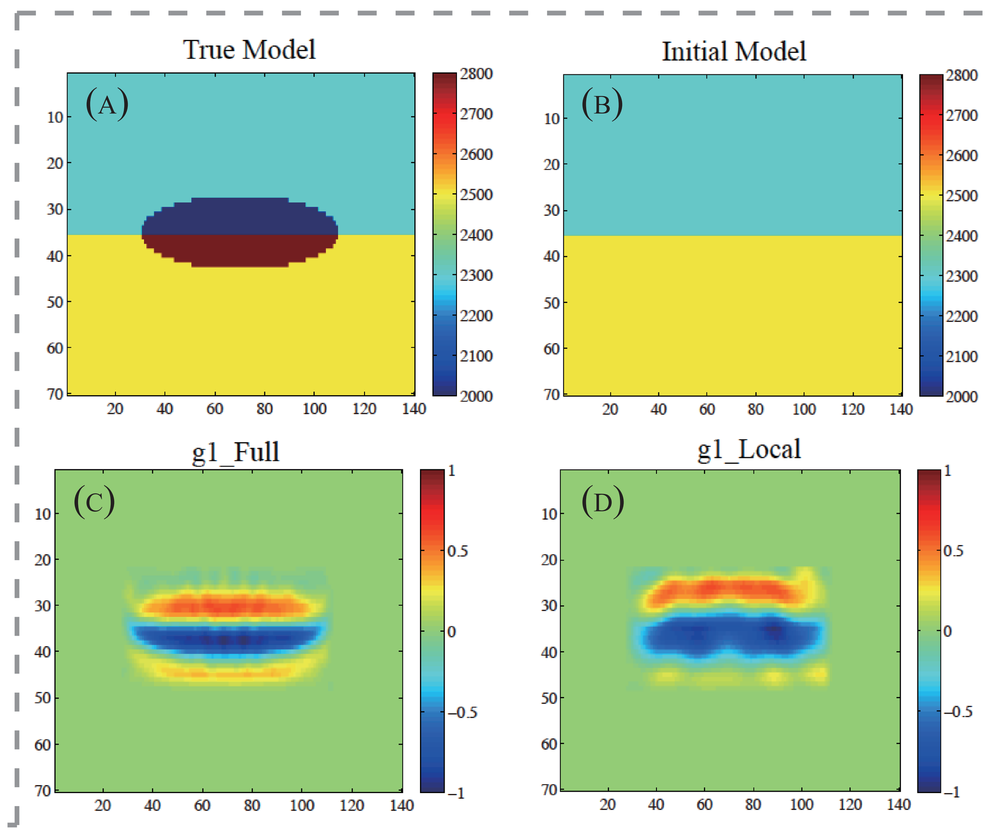



*Figure 5. The gradient of the first iteration. (A) The true P wave velocity model after perturbation. (B) The baseline/initial P wave velocity model for localized FWI. (C) The first P wave velocity gradient using full model simulation. (D) The first P wave velocity gradient using localised FWI.*

## Conclusions

1. Wavefield injection is robust enough for localized forward modeling;
2. The misfit in the wavefield extrapolation is caused by the truncated integral. This problems can be partially solved by well distributed shots;
3. Localized FWI works well based on gradient result during the first iterative process;
4. Localized FWI can reduce computational cost to a large extent.

## References

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3. Ravasi, M., & Curtis, A. (2013). Elastic imaging with exact wavefield extrapolation for application to ocean-bottom 4C seismic data.
4. Komatitsch, D., & Martin, R. (2007). An unsplit convolutional perfectly matched layer improved at grazing incidence for the seismic wave equation.