Evolution of shock waves

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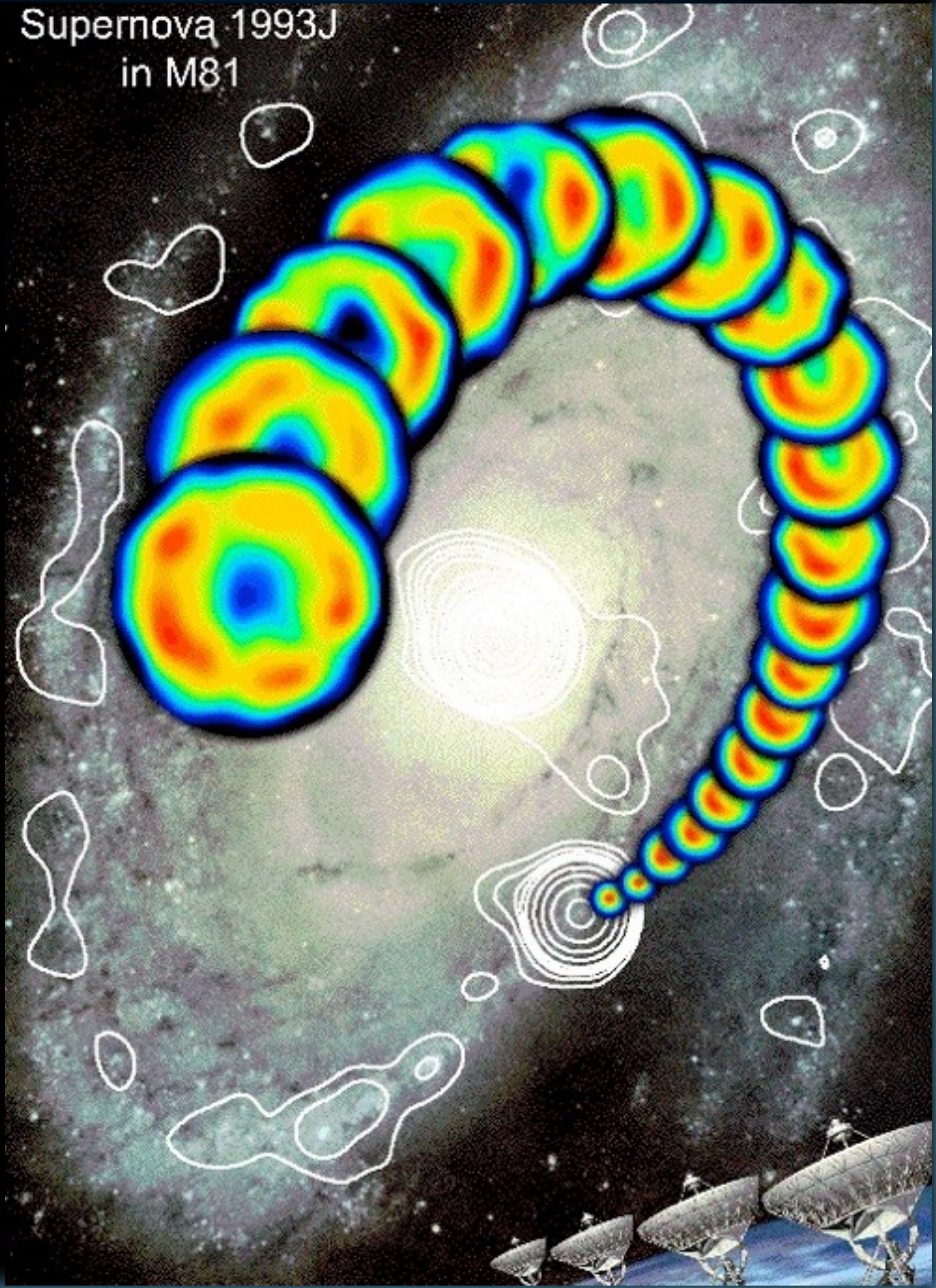
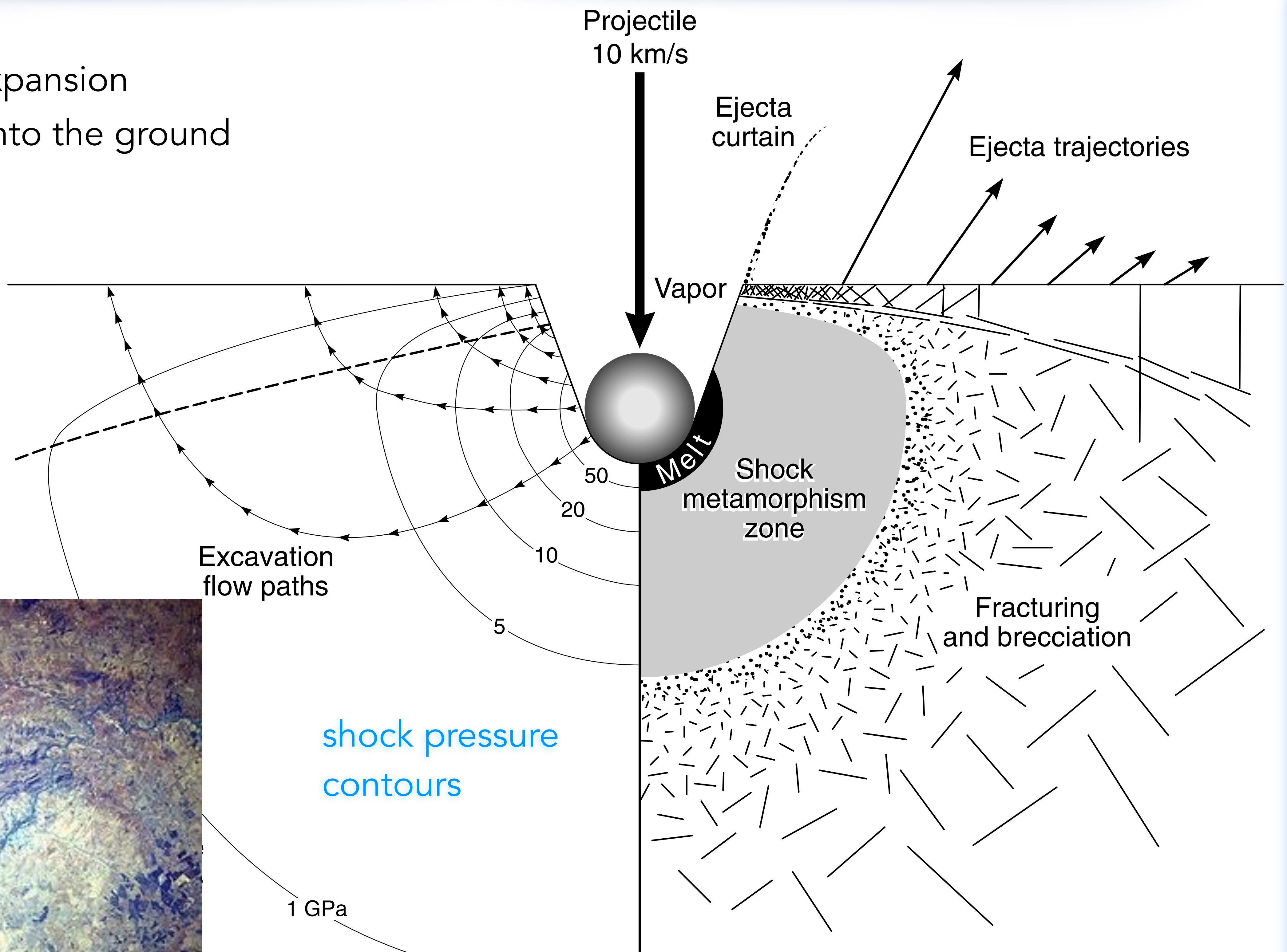
- different evolutionary phases
  - ★ free expansion: bullet
  - ★ adiabatic expansion (Sedov): constant energy
  - ★ snow-plough expansion: constant momentum



impact crater formation

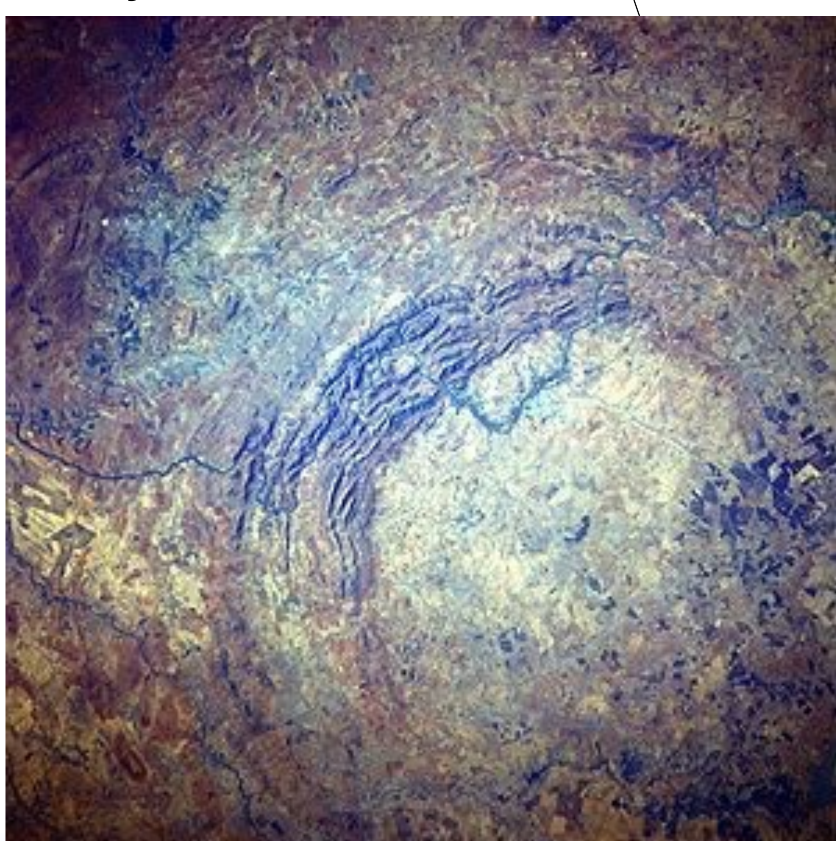
UnivEarthS

- quasi-spherical expansion of a shock wave into the ground



shock pressure contours

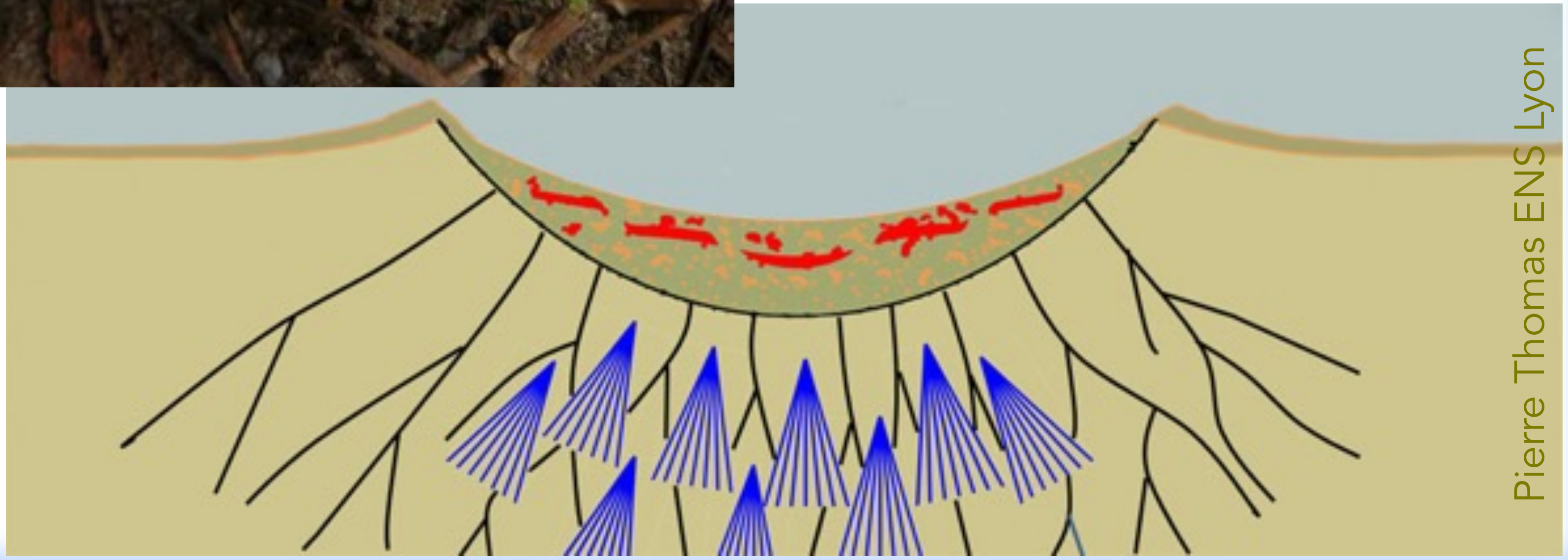
- Vredefort crater
  SouthAfrica
  200 miles wide
  2Gyr old



http://www.lpi.usra.edu/publications/books/CB-954/chapter3.pdf

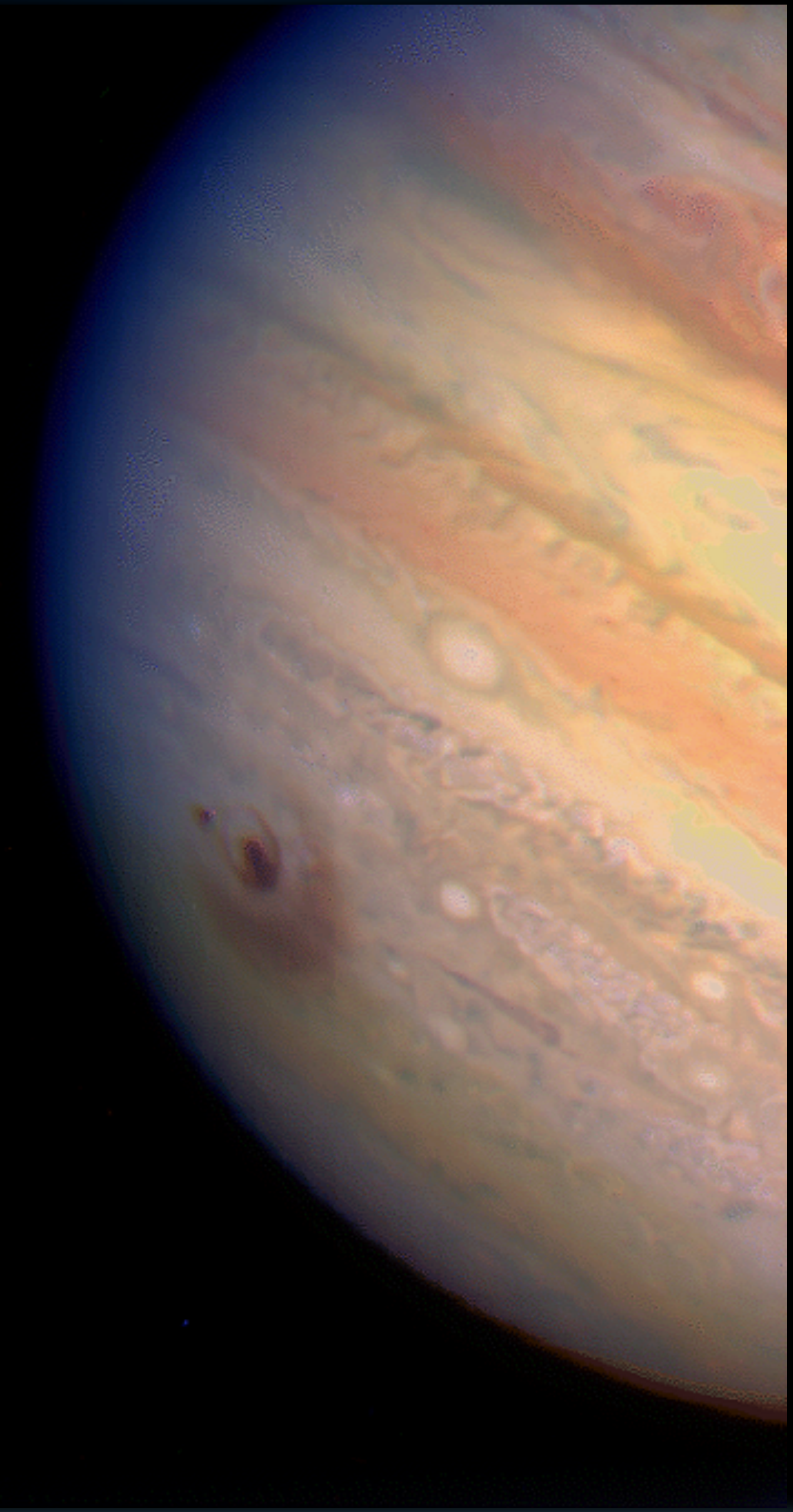
- shatter-cone striations
- systematic increase of striation angles with distance from the impact
- result from nonlinear waves (front waves) that propagate along a fracture front

Sagy, Reches, Fineberg 2002,
Nature 418, 310-313



comet impacts

- Shoemaker-Levy 9 on July 18, 1994
  45 mn after impact at 60 km/s
  waves visible for weeks



- Chicxulub impact, Yucatan
  KT-transition 66 Myr ago

forward & return shocks

- ejecta

  density profile often as power laws => self-similar solutions $\rho_{\mathbf{ej}} \propto \mathbf{r}^{-\mathbf{n}}$

  piston into interstellar medium (ISM) => forward shock gains mass and slows down

- shocked ISM = piston on ejecta => return shock to convert the ram energy of ejecta to pressure and to slow them down

- contact discontinuity between the two media

  ✦ $v_e \sim v_p \sim v_{shock} \Rightarrow m_p v_p \gg m_e v_e \Rightarrow \mathbf{T_p >> T_e}$

  ✦ electrons heat up by Coulomb diffusion

  $\Rightarrow$ delay $\tau_{Coulomb}$ (if no plasma effetcs)

  ✦ velocity of forward shock (FS) ≠ velocity of return shock (RS)

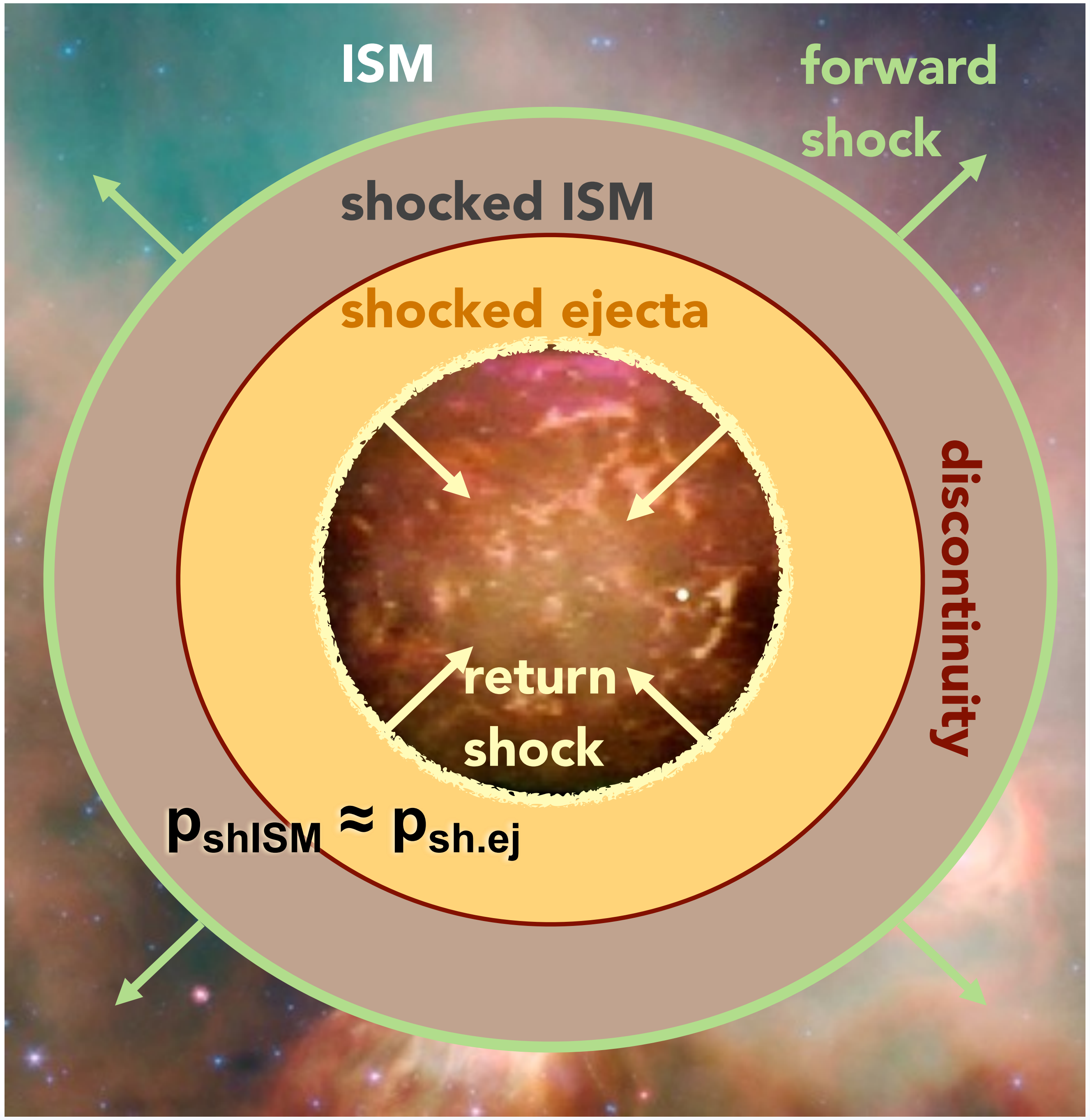
  $\Rightarrow$ **T(shocked ISM) ≠ T(shocked ejecta)**

  ✦ but **p(shocked ISM) ≈ p(shocked ejecta)** because subsonic velocities behind shocks

  $\Rightarrow$ densities (ideal gas):

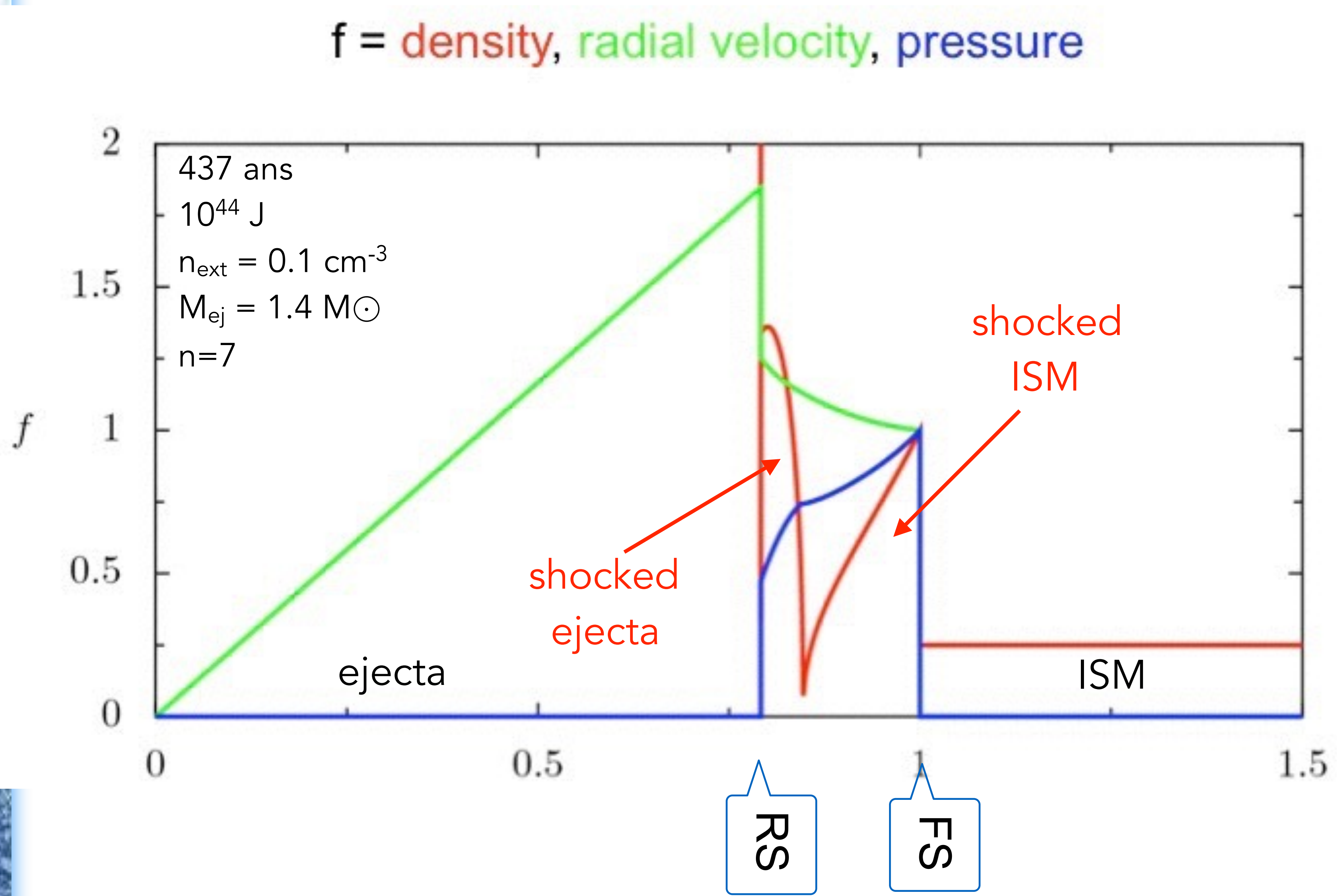
  n(shocked ISM) ≠ n(shocked ejecta)

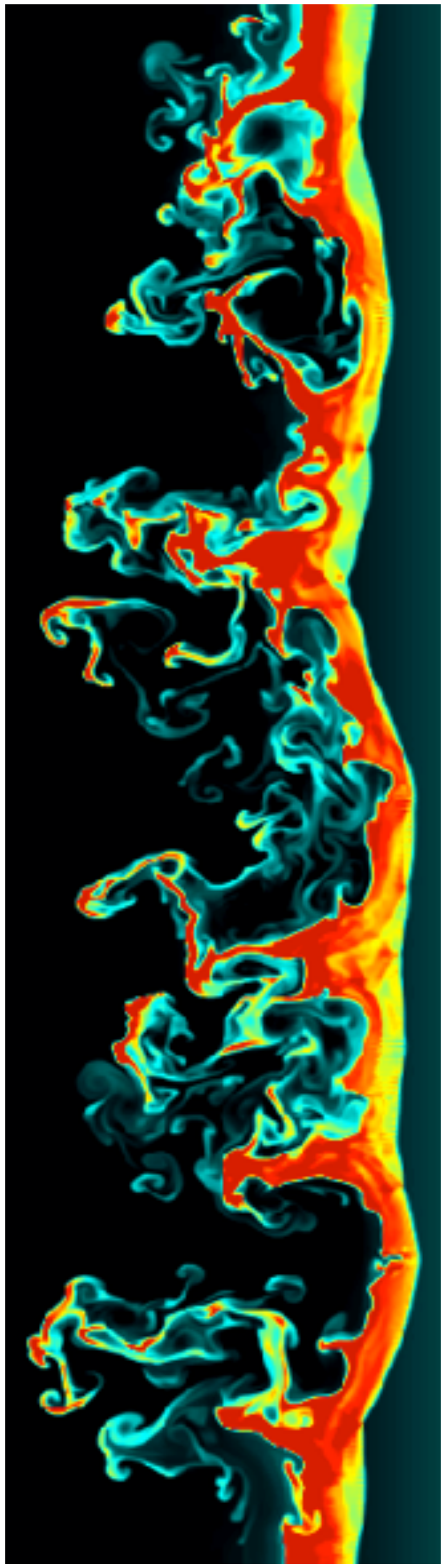
  ✦ RS moves inward wrt FS, but outward wrt the outside observer



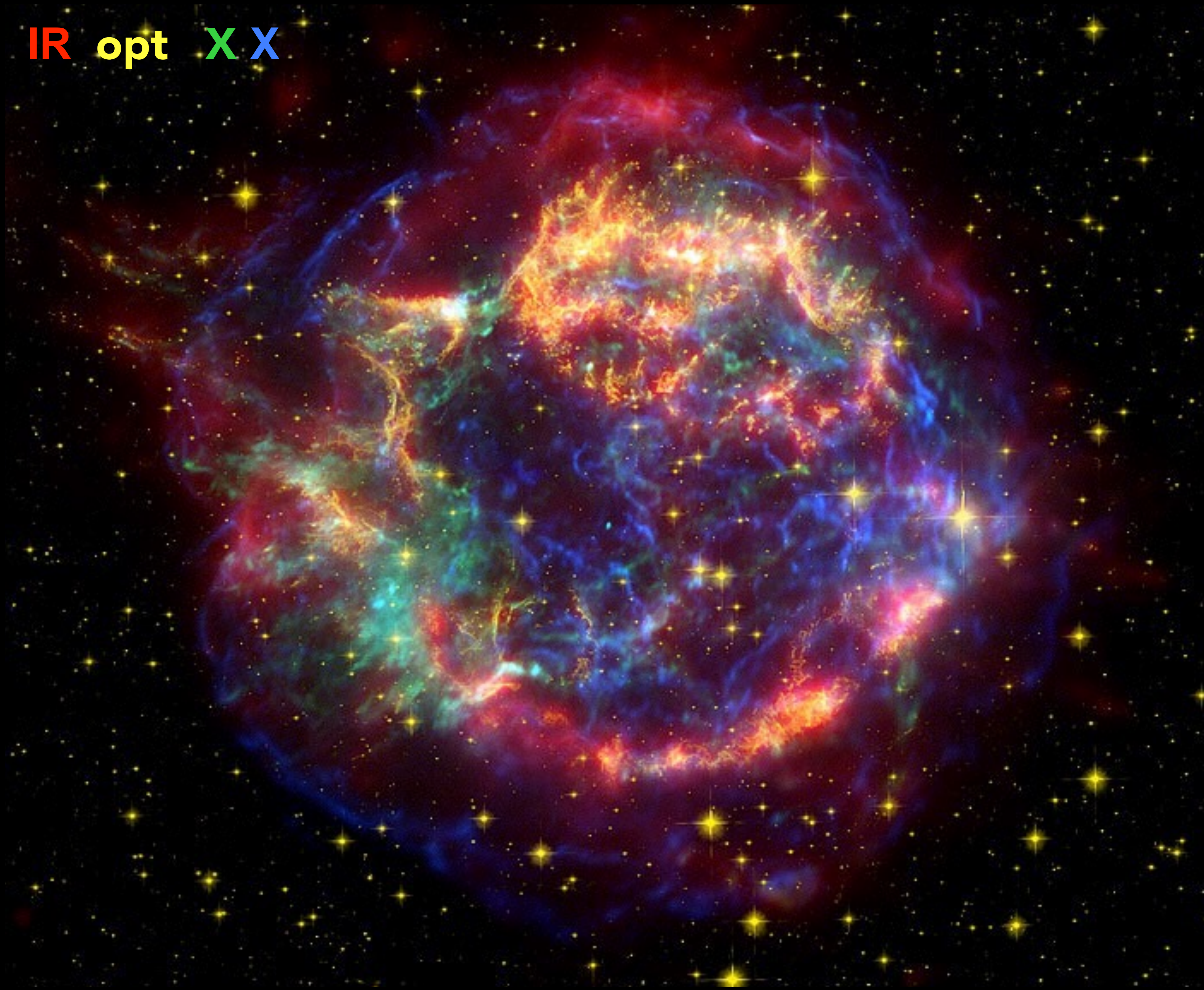
forward & return shocks

- density profiles: compression at shock, rarefaction behind expanding shock



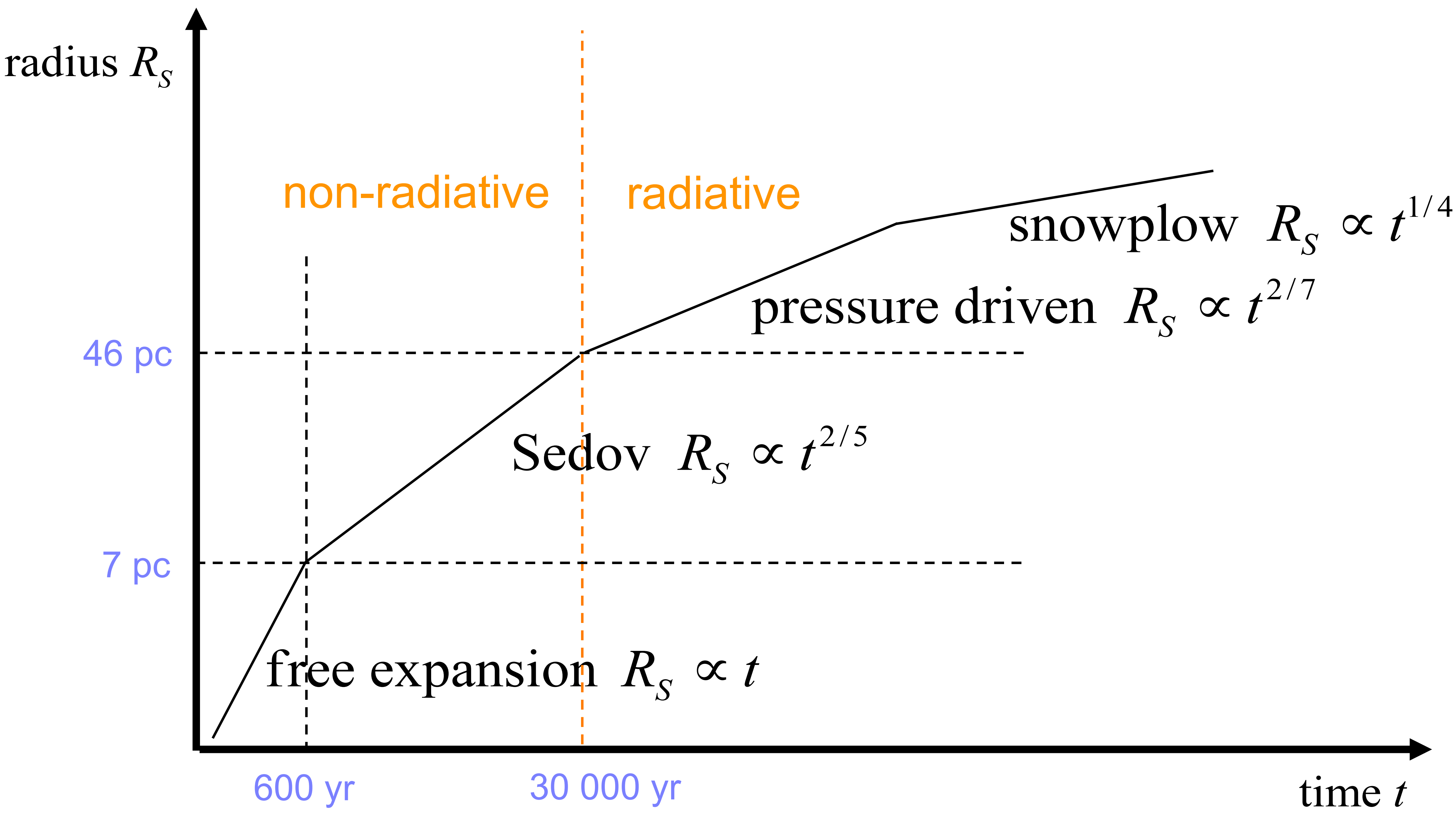


Cassiopeia A



expansion phases

- ex: E_{SN} ~ 10^{44} J supernova with 1.4 M⊙ of ejecta expanding into ISM with n = 0.1 H cm^{-3}



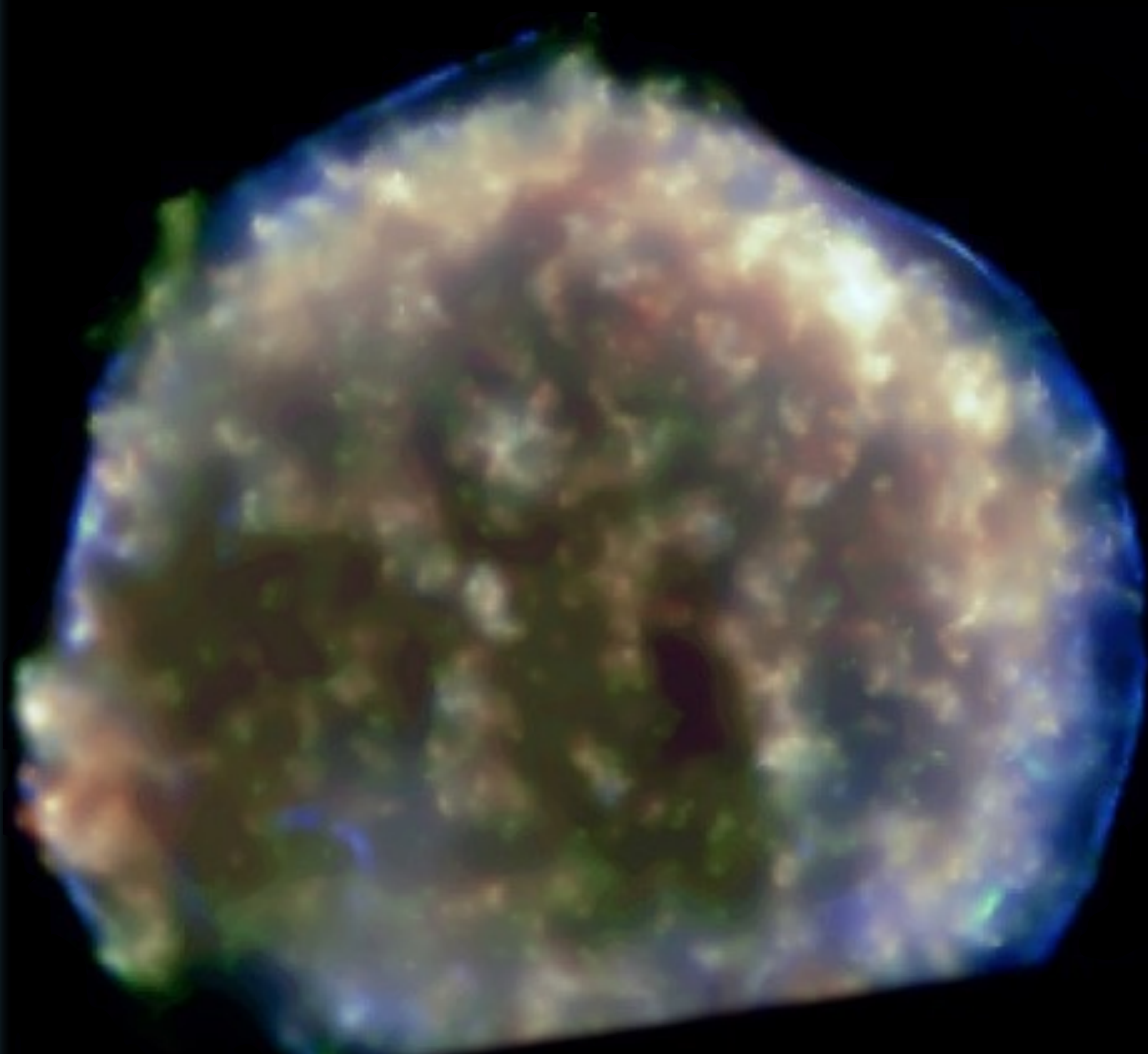
free expansion

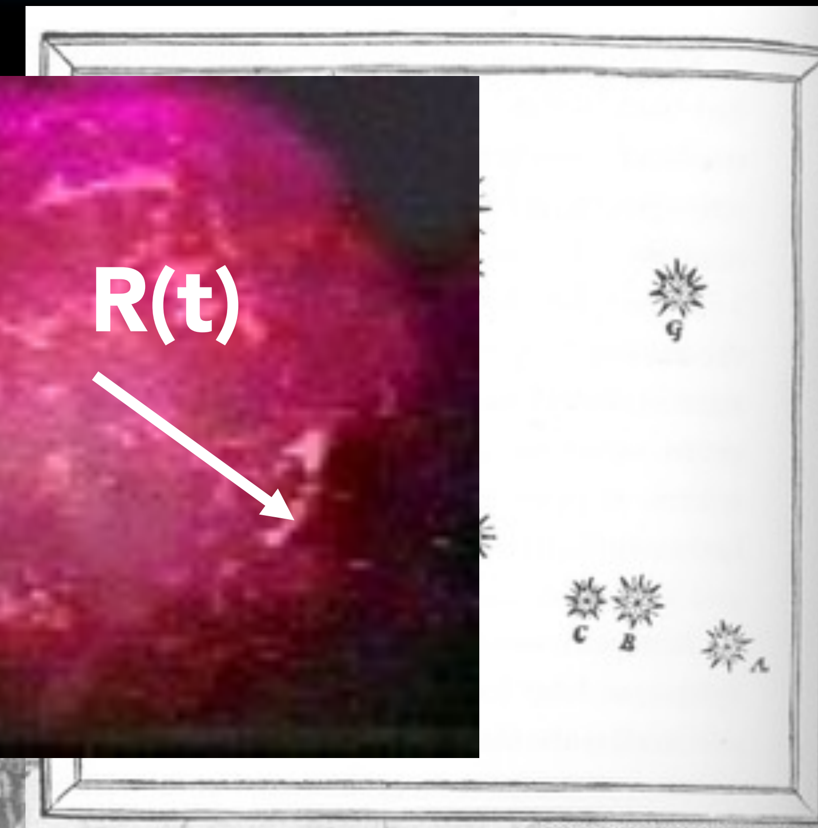
- free expansion: bullet mode

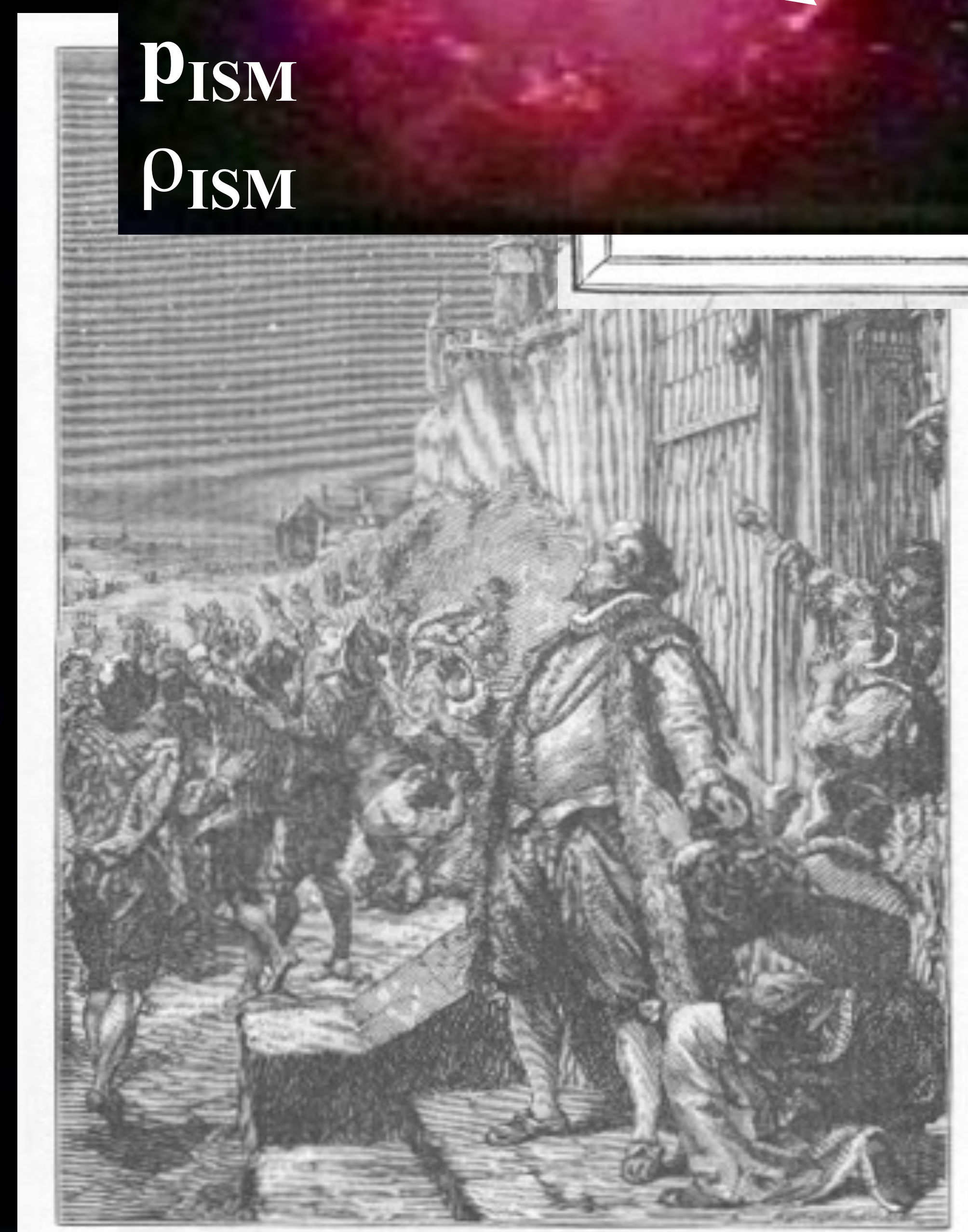
$$\mathbf{M_{ej} \gg \frac{4}{3}\pi R^3 \rho_{ISM} \quad E_{SN} = \frac{1}{2} M_{ej} v_{ej}^2}$$

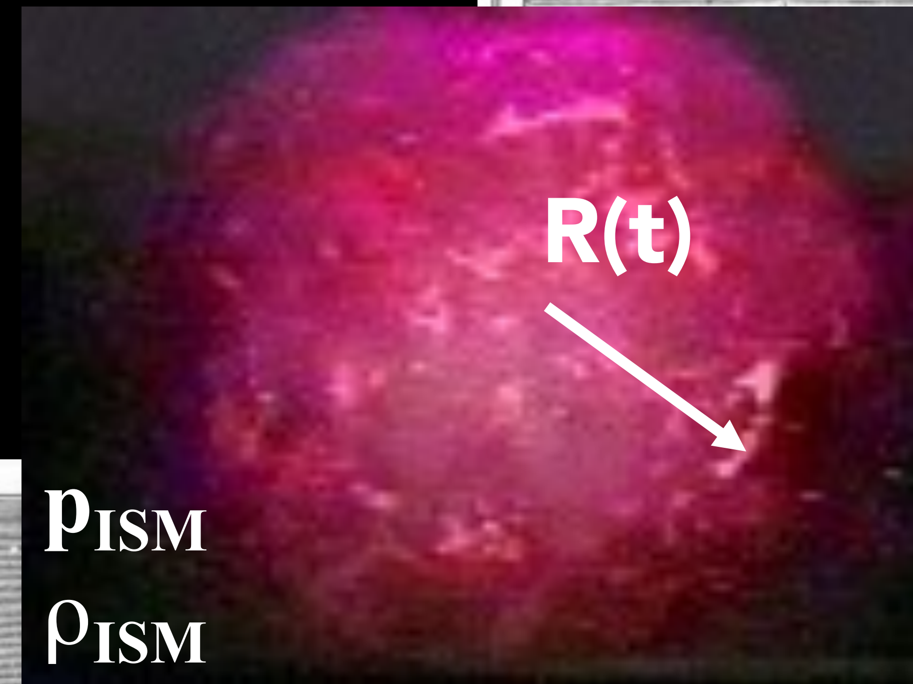
$$\mathbf{\dot{R} \approx cte = v_s \Rightarrow R = v_s t}$$

- as long as M(ejecta) ≫ M(swept up ISM)
- Tycho: 1572





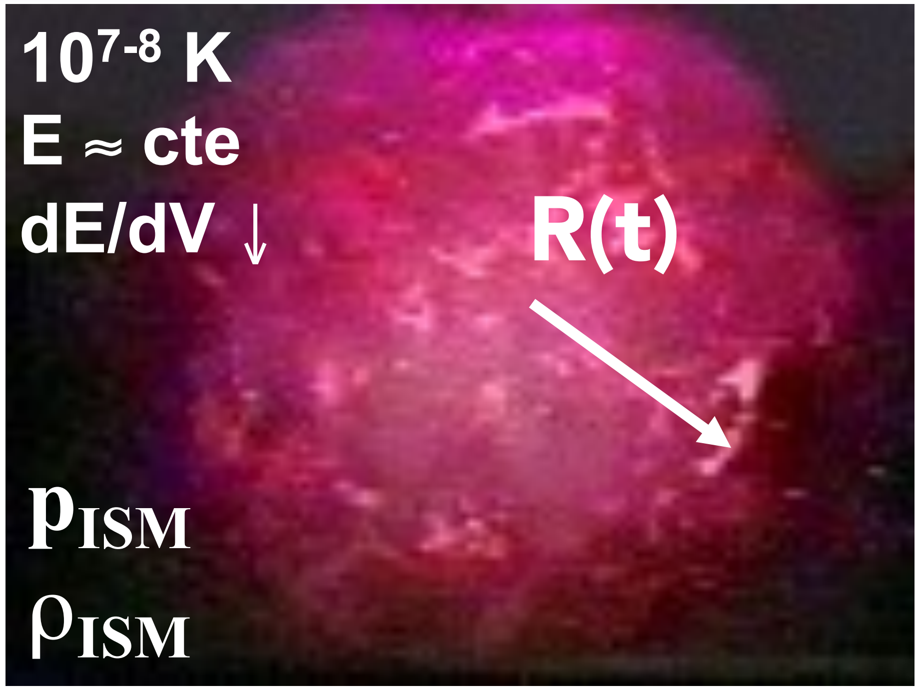




adiabatic expansion

- Sedov phase, self-similar solutions
  - ✦ M(swept up ISM) » M(ejecta) and p(ISM) « p(shocked ISM)

    thus role of M(ejecta) and p(ISM) «
- adiabatic conditions as long as
  - ✦ $\dot{E}_{rad}$ « $E \approx E_{SN}$ » E(swept up mass)
- thus evolution controlled only by E_{SN} et ρ_{ISM}
  - ✦ dimensional considerations



$$[\mathbf{E}] = [\mathbf{M}][\mathbf{L^2}][\mathbf{T^{-2}}] \Rightarrow \mathbf{E} \propto \mathbf{M}\,\mathbf{R^2 t^{-2}}$$

$$\mathbf{M} = \frac{4}{3}\pi \mathbf{R^3} \rho_{\mathbf{ISM}} \Rightarrow \mathbf{E} \propto \rho_{\mathbf{ISM}} \mathbf{R^5 t^{-2}} \Rightarrow \mathbf{R_s} = \xi \left(\frac{\mathbf{E}}{\rho_{\mathbf{ISM}}}\right)^{1/5} \mathbf{t^{2/5}}$$

- indeed if $\Phi \approx 1$ gathers complexity (inhomogeneities, non-sphericity, etc…)

$$\mathbf{E_{SN}} = \Phi \mathbf{M_{sh.ISM}} \left(\frac{\mathbf{dU_{sh.ISM}}}{\mathbf{dm}} + \frac{\mathbf{dE_{kin.sh.ISM}}}{\mathbf{dm}}\right) = \Phi \rho_{\mathbf{ISM}} \frac{4}{3}\pi \mathbf{R^3} \left(2 \times \frac{9}{32}\dot{\mathbf{R}}^2\right) = \Phi \rho_{\mathbf{ISM}} \frac{4\pi}{3} \mathbf{R^3} \dot{\mathbf{R}}^2$$

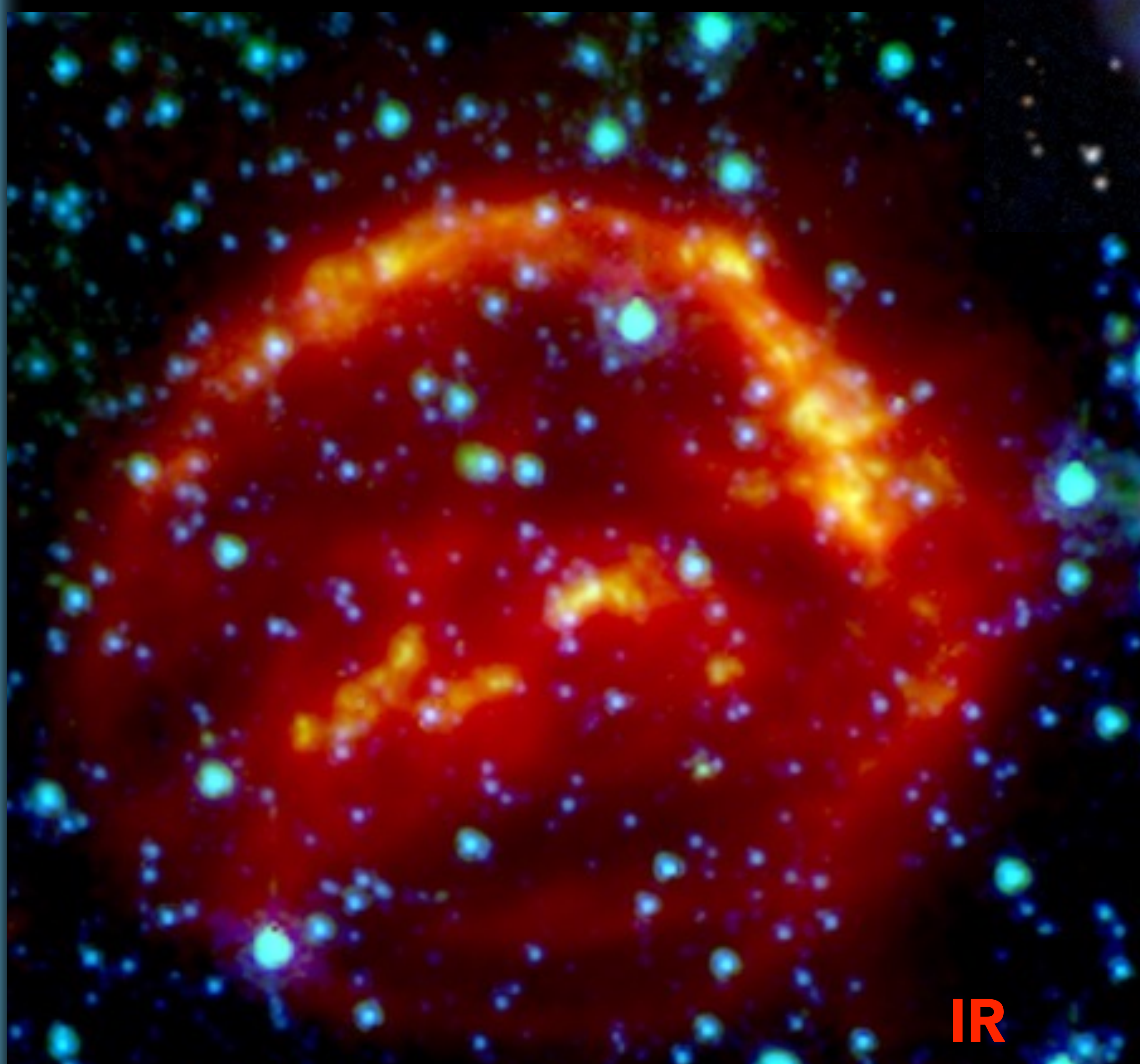
power-law solution $R(t) = a\, t^n$ => 3n + 2n - 2 = 5n - 2 = 0 => $t^{2/5}$

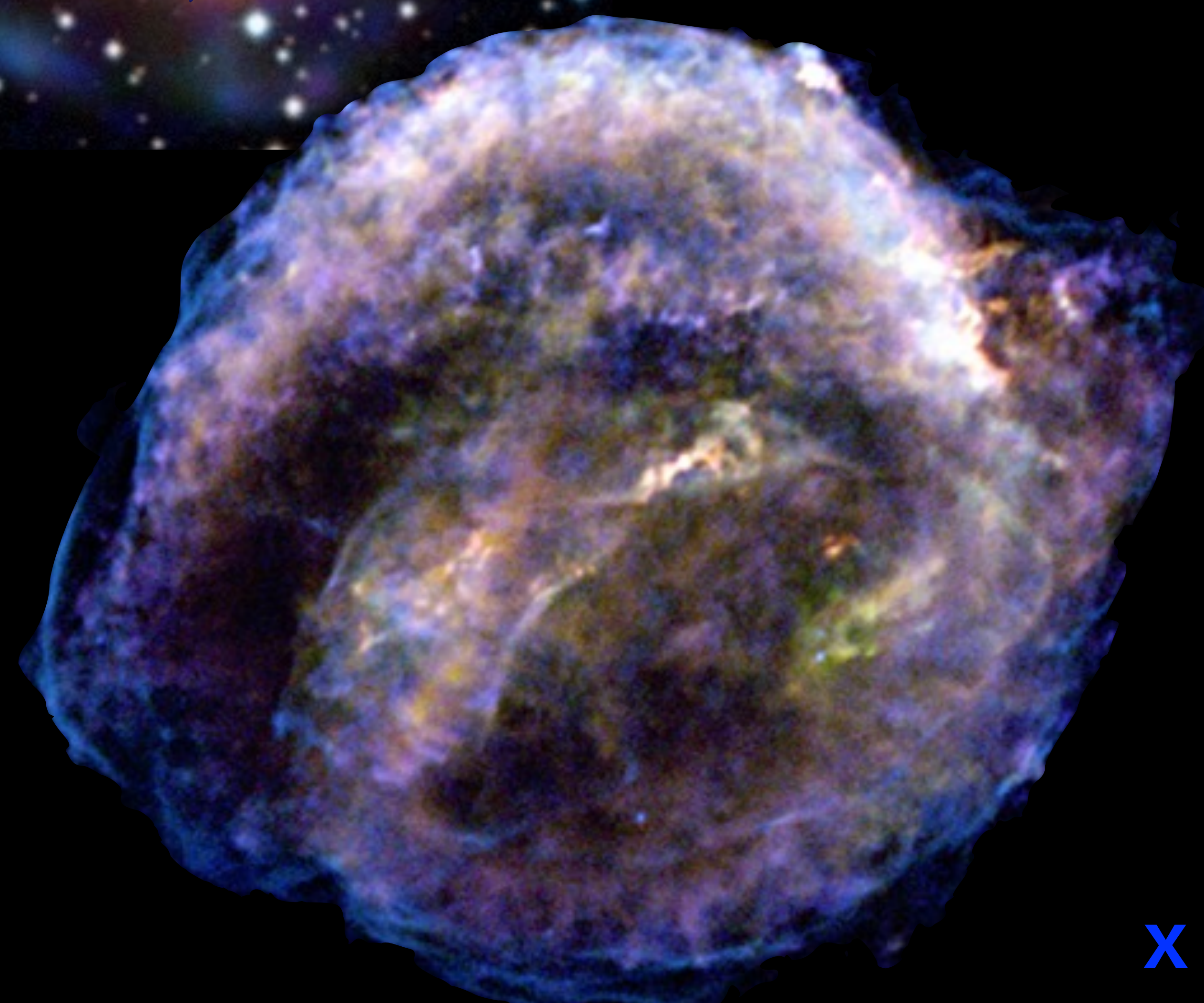
then the full equation becomes $E_{SN} = \Phi \frac{3\pi}{25} \rho_{MIS} a^5$ which yields the constant "a"

$$\mathbf{R} = \left(\frac{25}{3\pi\Phi}\right)^{1/5} \left(\frac{\mathbf{E_{SN}}}{\rho_{\mathbf{ISM}}}\right)^{1/5} \mathbf{t^{2/5}} \quad \text{and} \quad \mathbf{v_s} = \frac{2}{5}\left(\frac{25}{3\pi\Phi}\right)^{1/5} \left(\frac{\mathbf{E_{SN}}}{\rho_{\mathbf{ISM}}}\right)^{1/5} \mathbf{t^{-3/5}}$$

Kepler (9/10/1604)

- SNIa, free-Sedov transition





Sedov-radiative transition

- radiative losses: radiated power = $L_{rad} = n^2\,\Lambda(T)$
- at the end of Sedov:
  - ✦ downstream $T_2 >> 10^6$ K
    => radiative losses ≪, inefficient cooling
  - ✦ $T_2 \leq 10^6$ K efficient —> important losses

$$v_c \propto \left(\frac{E_{SN}}{\rho_{MIS}}\right)^{1/5} t^{-3/5}$$

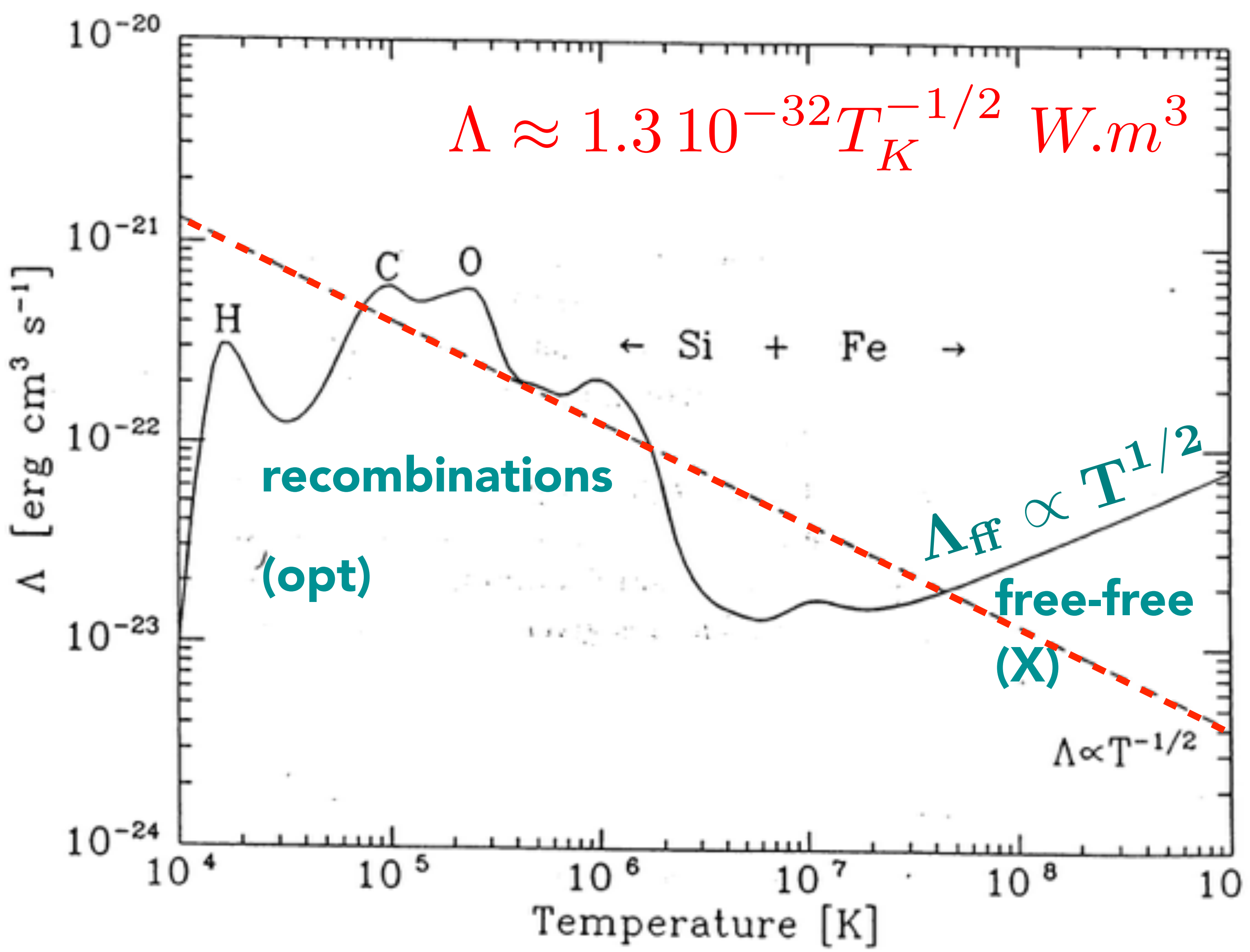
$$T_2 = \frac{\mu m}{k}\frac{3}{16} v_c^2 \quad \Rightarrow \quad T_2 \propto t^{-6/5} \downarrow\downarrow$$

- radiative phase starts when $\tau_{rad} < \tau_{expansion}$

$$\tau_{rad} = \frac{U_{MISch}}{L_{rad}} = \frac{3}{2}\frac{(n_i + n_e)kT_2}{n_i n_e \Lambda(T_2)}$$

$$= \frac{3kT_2}{n_2\Lambda(T_2)} = \frac{9\mu m v_c^2}{64 n_{MIS}\Lambda(T_2)}$$

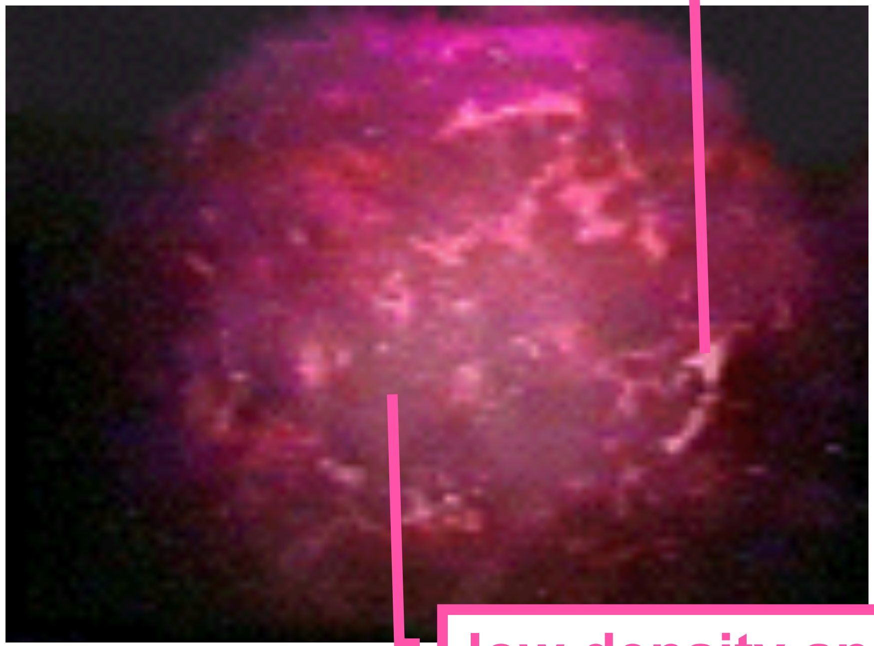
$$\tau_{exp} = \text{âge}$$



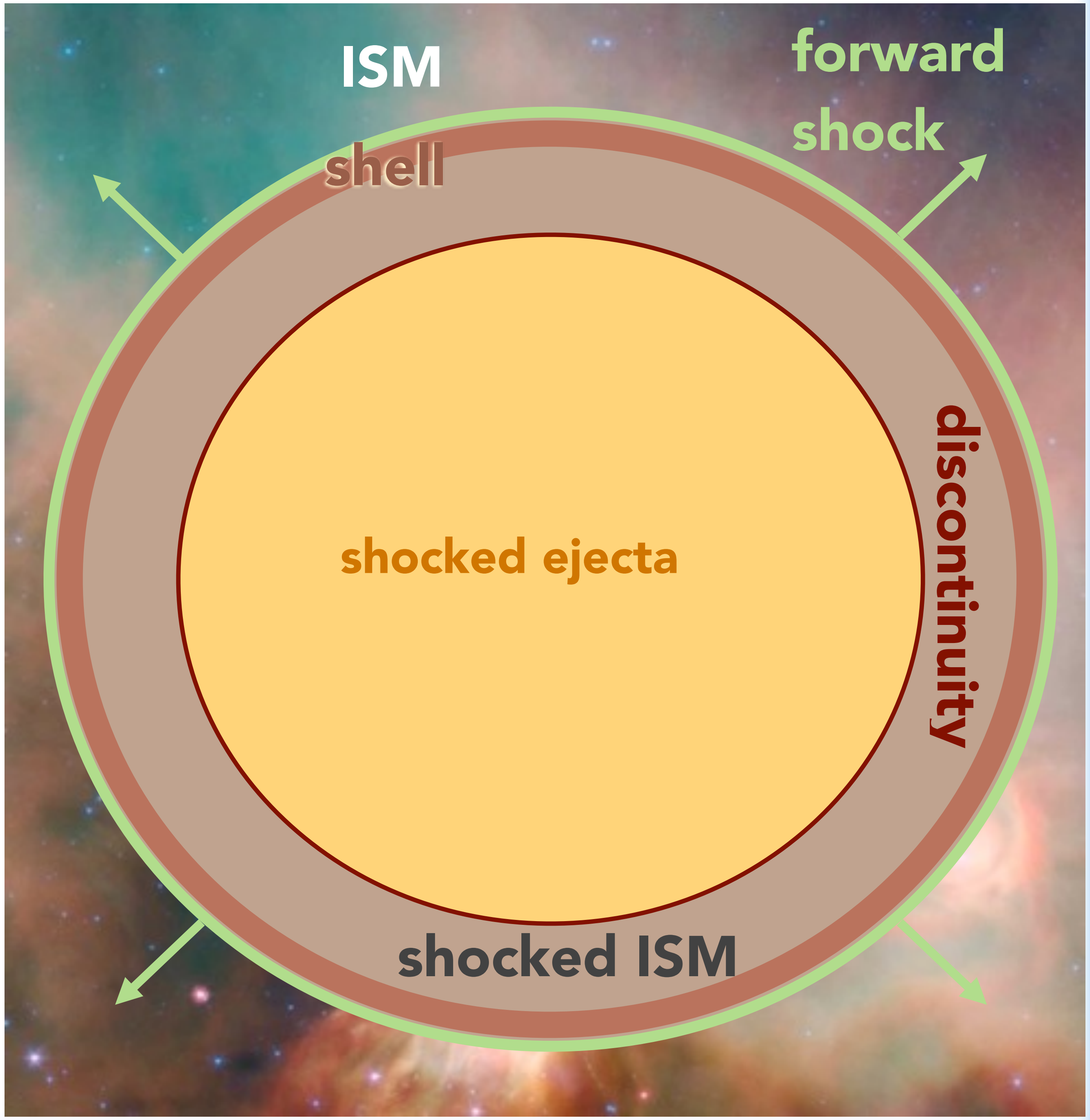
radiative phase

- radiative losses rapidly become catastrophic:
  because T↓ ⇒ n↑ ⇒ losses ↑↑
- => thin shell with dense filaments radiating mostly in the optical (electron recombination lines)

radiating filaments
200 km/s, < 10^6 K



low density and low
emission interior < 10^7 K



Cygnus loop

- filaments in the optical

radiative phase

- characteristic scales

$$R_{PDS} = 14\,\mathrm{pc}\,\zeta_{metal}^{-1/7}\left(\frac{E_{SN}}{10^{44}\,J}\right)^{2/7}\left(\frac{n_H}{10^6\,m^{-3}}\right)^{-3/7}$$

$$t_{PDS} = 13280\,\mathrm{yr}\,\zeta_{metal}^{-5/14}\left(\frac{E_{SN}}{10^{44}\,J}\right)^{3/14}\left(\frac{n_H}{10^6\,m^{-3}}\right)^{-4/7}$$

$$v_{PDS} = 413\,\mathrm{km/s}\,\zeta_{metal}^{3/14}\left(\frac{E_{SN}}{10^{44}\,J}\right)^{1/14}\left(\frac{n_H}{10^6\,m^{-3}}\right)^{1/7}$$

- only momentum conservation (since $E_{tot}\downarrow$)
- snow-plough phase: onset with a little pressure push from the hot interior gas where $n \sim 10^{-2\text{-}3}$ cm^{-3} and L_{rad} are still low and p_{int} ~ uniform acts as a piston on the thin radiative shell

with push

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_{ISM}\dot{R}\right) = 4\pi R^2 p_{int}$$

$$L_{rad} \ll \Rightarrow p_{int} V^{\gamma} = cte$$

$$\Rightarrow p_{int} \propto R^{-3\gamma} \propto R^{-5}$$

$$\Rightarrow \frac{d}{dt}\left(R^3\dot{R}\right) \propto R^2 R^{-5}$$

$$\text{solution as } t^n \Rightarrow 4n - 2 = -3n \Rightarrow n = \frac{2}{7}$$

$$R \propto t^{2/7} \quad \text{and} \quad \dot{R} = v_s \propto t^{-5/7}$$

later without push

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_{ISM}\dot{R}\right) = 0$$

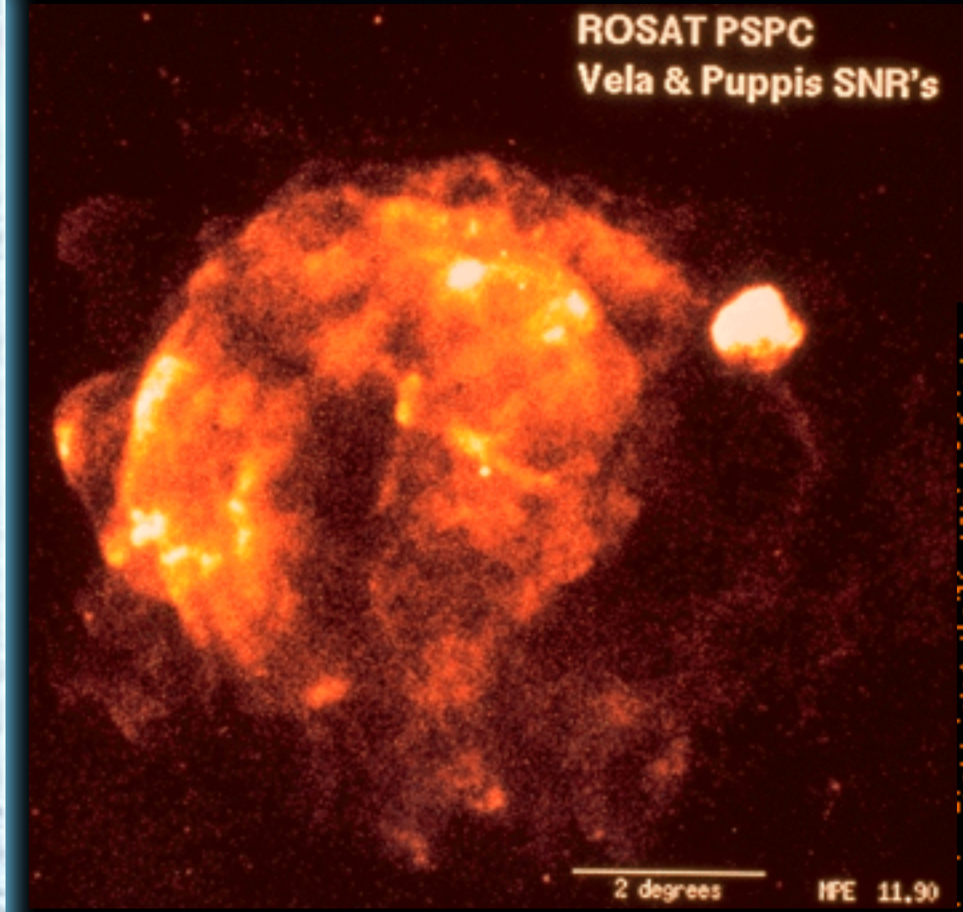
$$\Rightarrow R^3\dot{R} = cte = R_c^3\dot{R}_c$$

$$3n + n - 1 = 0 \Rightarrow R \propto t^{1/4}$$

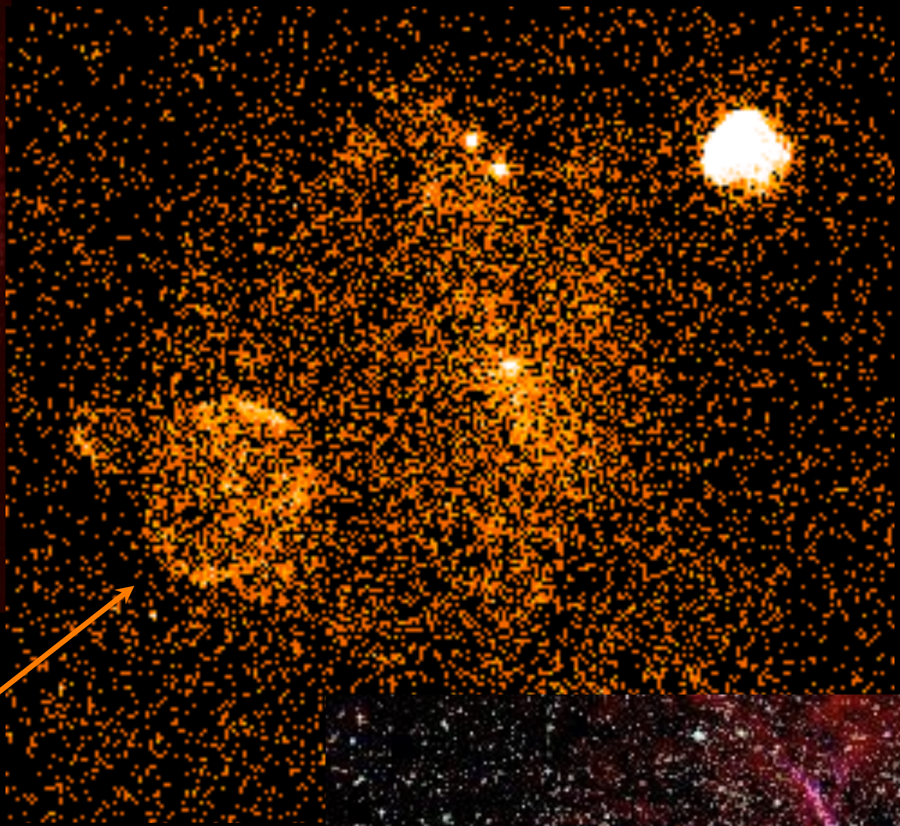
$$R = R_c\left[1 + \frac{4\dot{R}_c}{R_c}(t - t_c)\right]^{1/4}$$

$$\dot{R} = \dot{R}_c\left[1 + \frac{4\dot{R}_c}{R_c}(t - t_c)\right]^{-3/4}$$

Vela



soft X



hard X

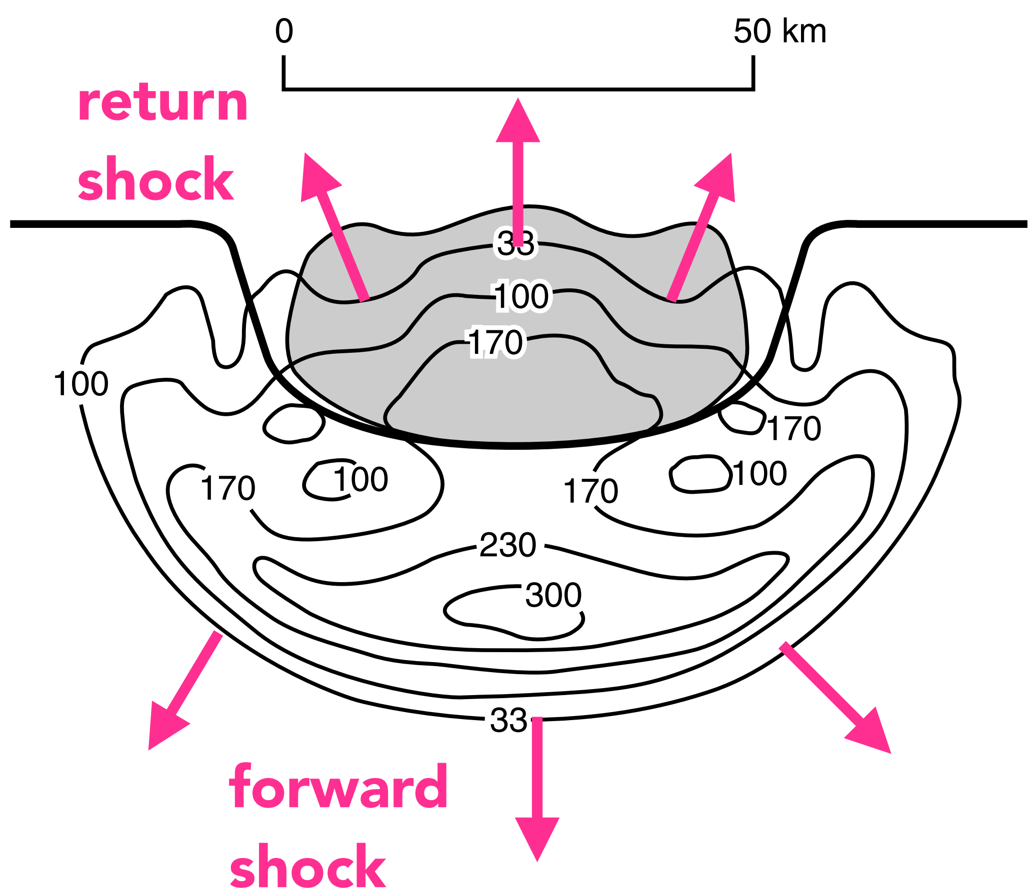
opt

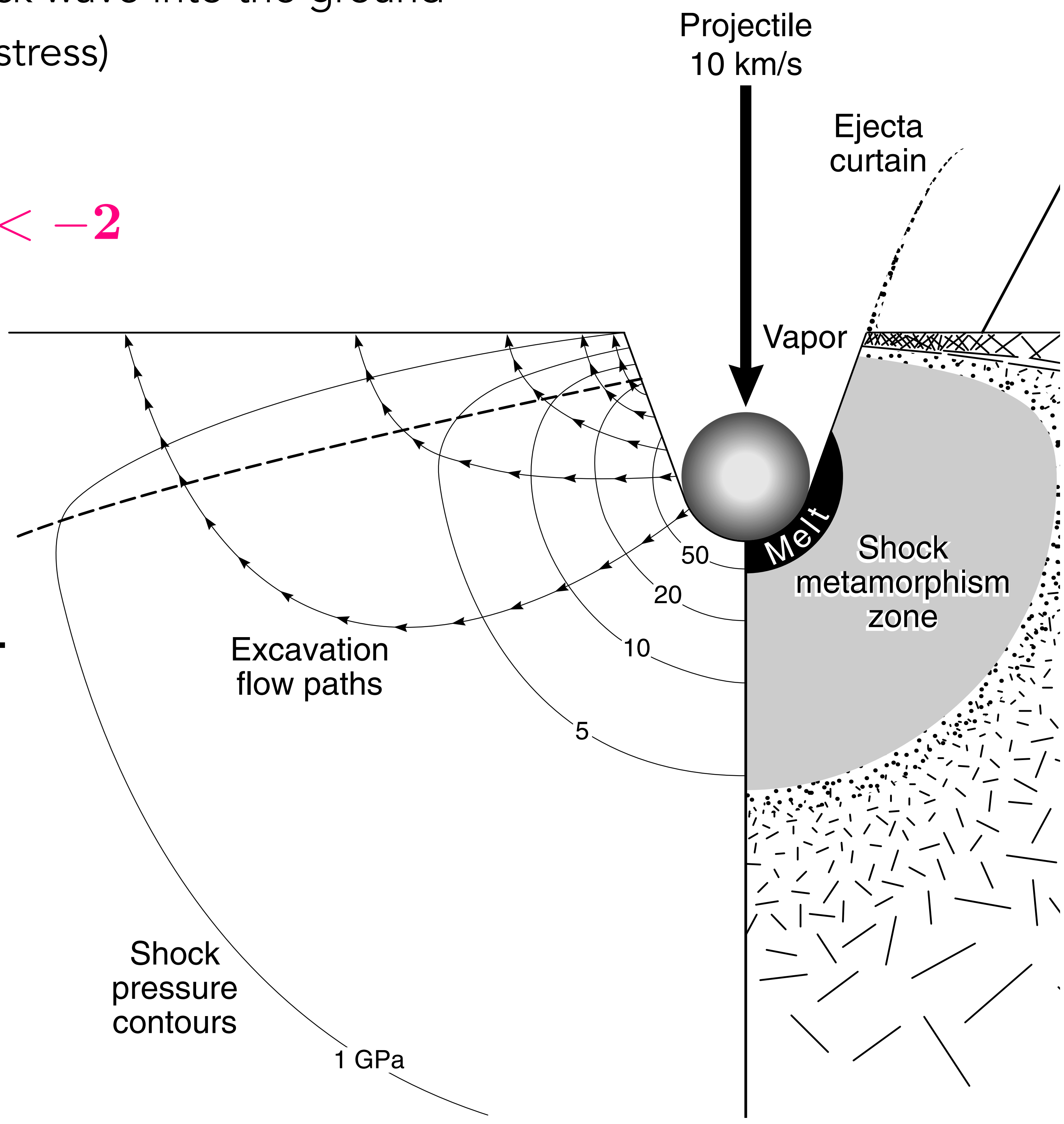
Vela
junior

- quasi-hemispherical expansion of a shock wave into the ground
- similar dimensional analysis (+gravity, +stress)

$$p_{sh.ground} \propto R^{-n}$$
$$\text{with} \quad -4.5 < n < -2$$

- return shock into the projectile
  => melting



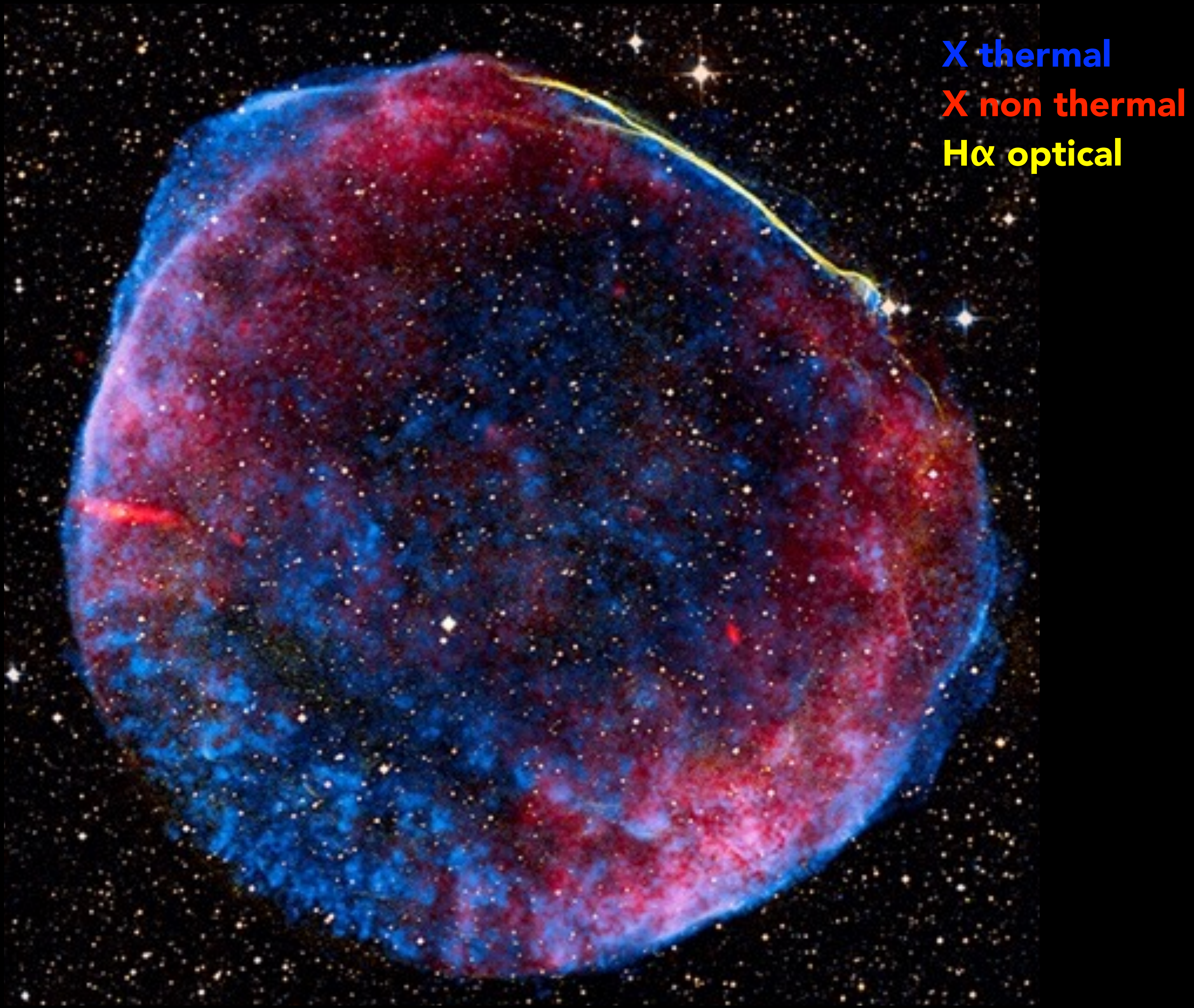


- the forward shock slows down and becomes a normal (elastic) seismic wave at c_{sound}


“isothermal” shocks

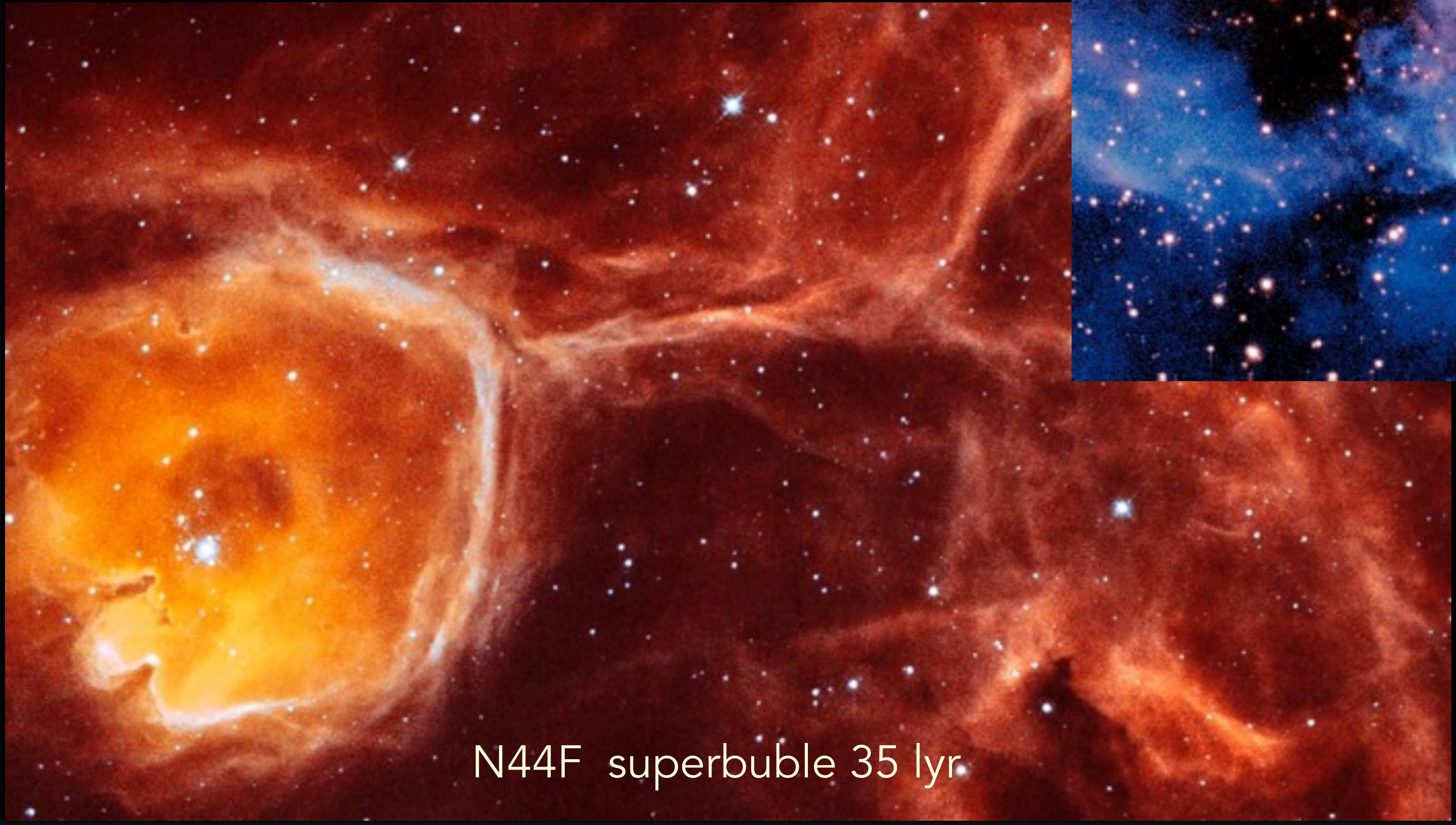
SN1006

- entering a dense cloud: slowing down and radiative filaments

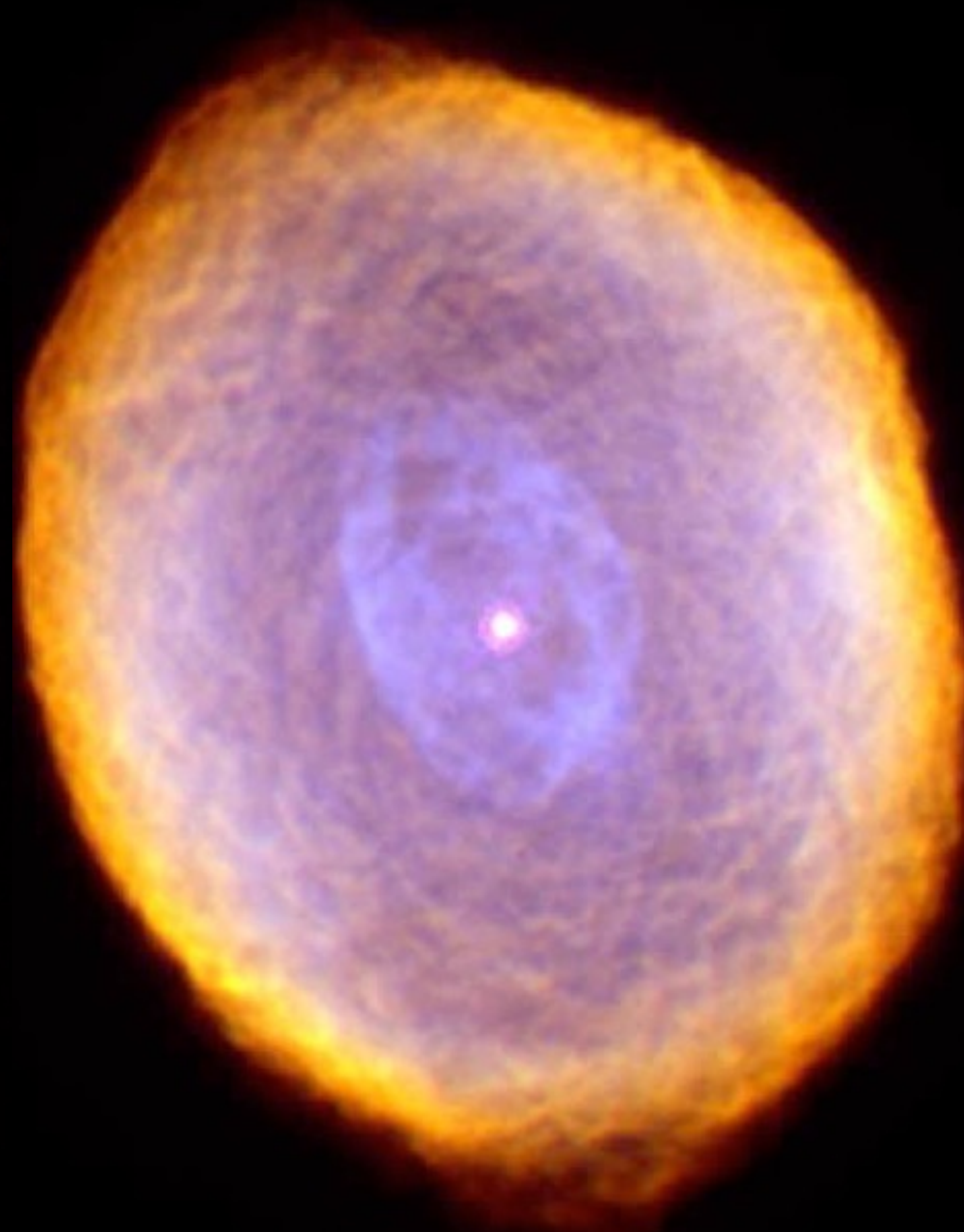


stellar winds

- $dM_w/dt \sim 10^{-5}\ M_\odot\ yr^{-1}$, > 1000 km/s
- radiation pressure driven
- single massive star
- stellar cluster

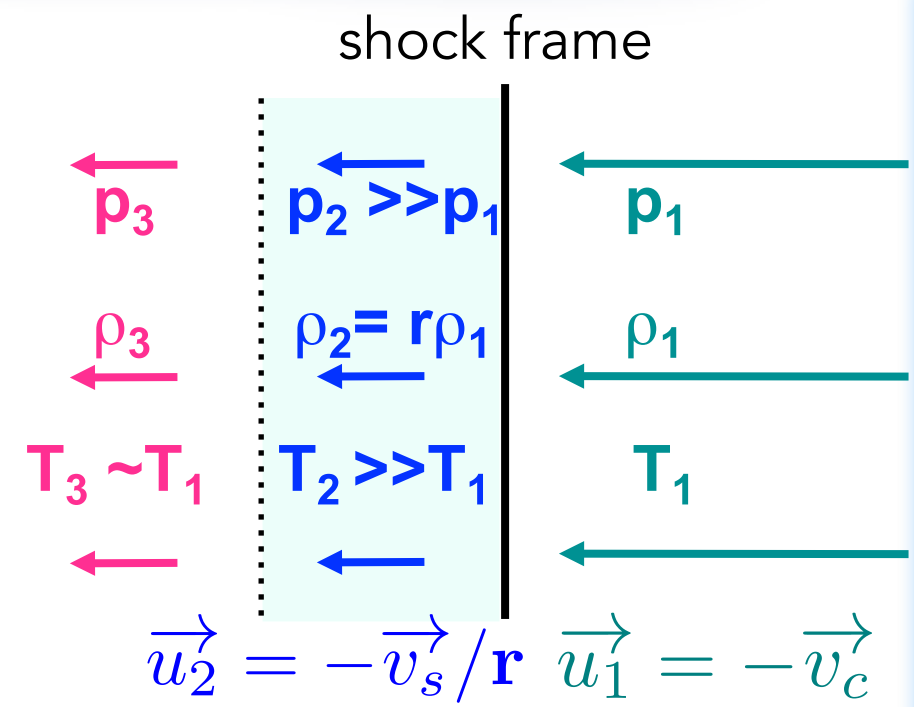


- $dM_w/dt \sim 10^{-8}\ M_\odot\ yr^{-1}$,
- radiation pressure driven



zoom on the radiative shock

- strong radiative losses => very thin radiative zone
  radiative cooling rate $\Lambda(T) \approx \Lambda_0 T^{-1/2}$ between 10^5 and 10^8 K
  width of the radiative zone = $\ell_{rad} = u_2 \tau_{rad}$



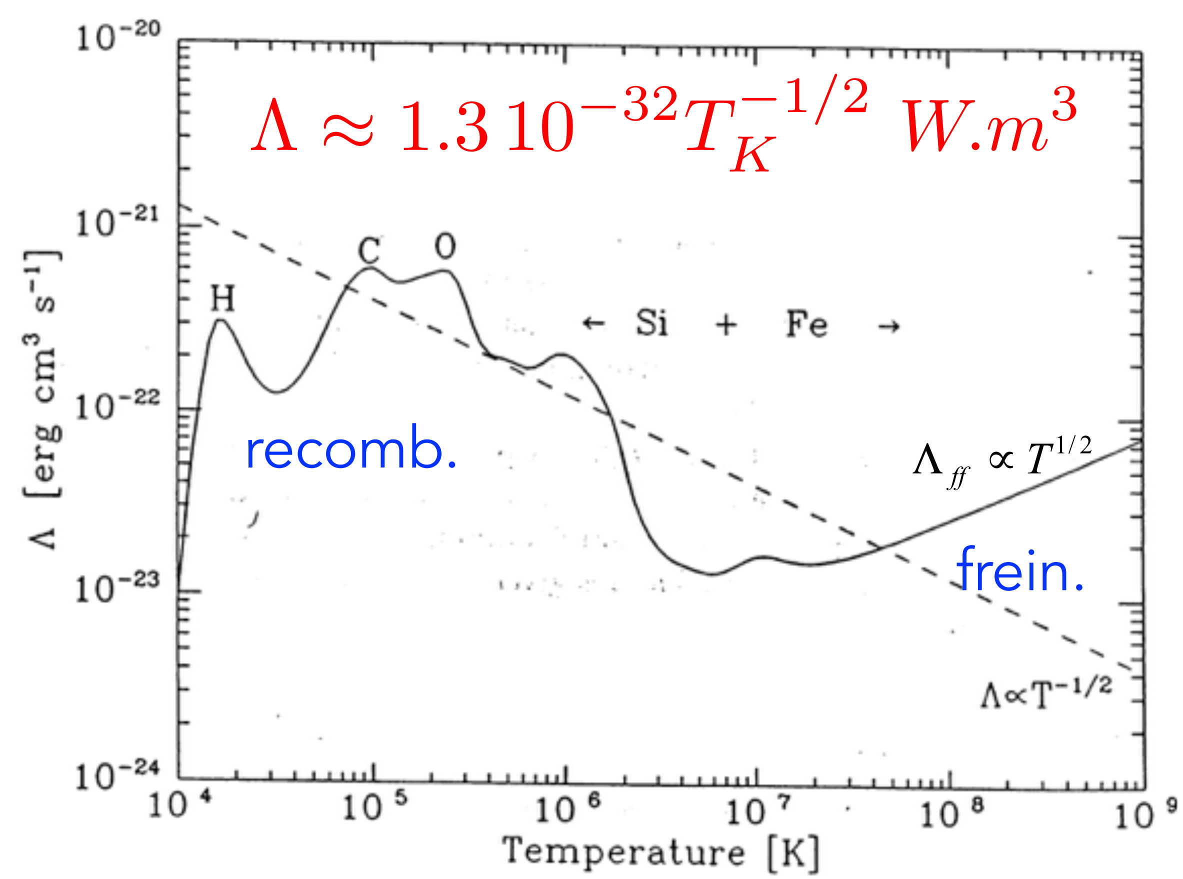
- radiative length scales with u_1^4:

$$\tau_{rad} = \frac{U_{MISch}}{L_{rad}} = \frac{3}{2}\frac{(n_i + n_e)kT_2}{n_i n_e \Lambda(T_2)} = \frac{3kT_2}{n_2 \Lambda(T_2)} = \frac{3kT_2^{3/2}}{r\, n_1 \Lambda_0}$$

$$T_2 = \frac{3\mu m}{16k} u_1^2 \quad \Rightarrow \quad \ell_{rad} = \left(\frac{3k}{r^2\, n_1 \Lambda_0}\right)\left(\frac{3\mu m}{16k}\right)^{3/2} u_1^4 \quad \propto u_1^4$$

- fast shock => ℓ_{rad} >> system size
  ⇒ adiabatic shock (Sedov)

- slow shock => very thin shell,
  downstream heat radiated away and the
  shock becomes "isothermal" because $T_3 \approx T_1$
  the bulk incident kinetic energy is thermalised
  by the shock and quickly radiated away



isothermal shock

- stationary shock without magnetic field support

  $T_3 \approx T_1$ maintained by radiative losses (and cosmic-ray acceleration)

  $p \propto \rho \quad (\gamma=1)$

  Mach number $\quad M_1 = \dfrac{u_1}{c_s}$

$$\boxed{\begin{aligned}&\text{masse :} \quad \rho_1 u_1 = \rho_3 u_3 \\ &\text{mvt :} \quad p_1 + \rho_1 u_1^2 = p_3 + \rho_3 u_3^2 \\ &T_1 = T_3 \Rightarrow c_{s1} = c_{s3} \Rightarrow p_1/\rho_1 = p_3/\rho_3\end{aligned}}$$

- compression ratio for a strong shock ($\rho_1 u_1^2 >> p_1$)

$$\text{mvt : } \rho_1 u_1^2 = p_3 + \rho_3 u_3^2 \quad \text{et} \quad \text{masse} \quad \rho_1 u_1 = \rho_3 u_3 \quad \Rightarrow \quad u_3^2 - u_3 u_1 + c_s^2 = 0$$

$$\text{racines : } u_3 = \frac{u_1}{2}\left[1 \pm \left(1 - \frac{4c_s^2}{u_1^2}\right)^{1/2}\right] = \frac{u_1}{2}\left[1 \pm \left(1 - \frac{4}{M_1^2}\right)^{1/2}\right] \approx \frac{u_1}{2}\left[1 \pm \left(1 - \frac{2}{M_1^2}\right)\right]$$

$$\text{racine triviale : } u_3 \approx u_1$$

$$\boxed{u_3 = \frac{u_1}{M_1^2} \Rightarrow \quad r = \frac{u_1}{u_3} = \frac{\rho_3}{\rho_1} = \frac{p_3}{p_1} = M_1^2}$$

- extreme compression not limited by the thermal energy which is quickly radiated away

$$p_3 = \rho_1 u_1^2 - \rho_3 u_3^2 = \rho_1 u_1^2 \left(1 - \frac{1}{r}\right) = \rho_1 u_1^2 \left(1 - \frac{1}{M_1^2}\right) \approx \rho_1 u_1^2$$

stellar wind bubble

- expansion speed v_{exp} ~ 20-100 km/s
- "isothermal" forward shock: T(shocked ISM) ≈ T(outside ISM) ≈ 10^4 K
- adiabatic return shock (standing shock)

  c_s(shocked wind) ~ 500 km/s >> v_{exp} => subsonic

  so nearly isobaric conditions

  ⇒ p(shocked ISM) ≈ p(shocked wind) = p

- p = d(mv)/dtdS flux of the wind

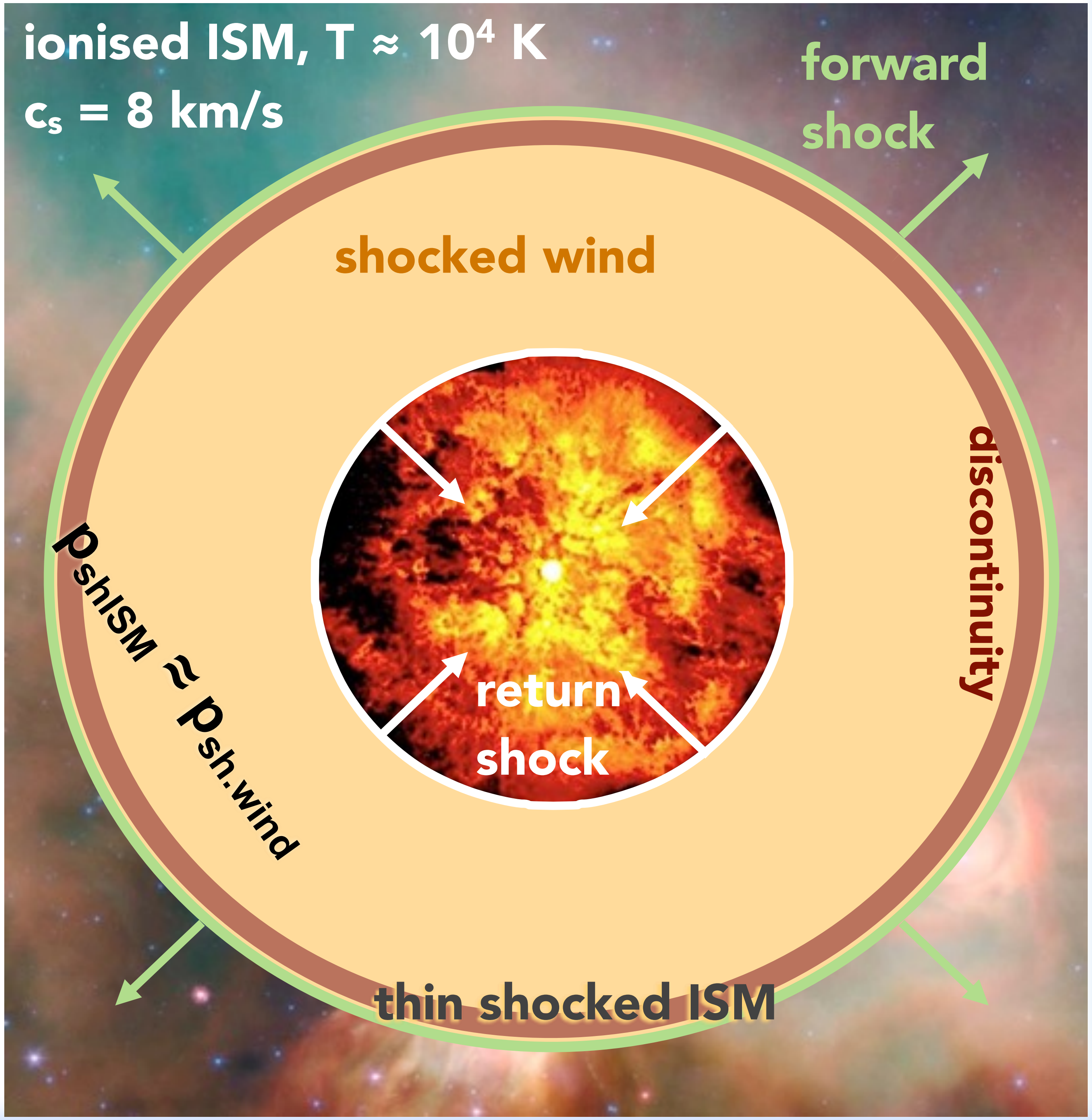
  at the level of the return shock (RS)

  in the upstream frame

$$\mathbf{v_{shocked\ wind}} = \left(1 - \frac{1}{r}\right) u_1$$

$$= \left(1 - \frac{1}{r}\right) v_w$$

$$p = \frac{1}{4\pi R_{RS}^2} \dot{M}_w \left(1 - \frac{1}{r}\right) v_w$$



- simplifications: the hot gas occupies a constant fraction Φ of the volume

$$\Phi = 1 - \left(\frac{R_{RS}}{R_{FS}}\right)^3$$

- pushed snow-plough, pushed by the common pressure p of the shocked wind and ISM

$$\frac{d}{dt}\left(\frac{4}{3}\pi R^3 \rho_{HII} \dot{R}\right) = 4\pi R^2 p \quad \Rightarrow \quad \frac{p}{\rho_{HII}} = \dot{R}^2 + \frac{1}{3} R\ddot{R} \qquad (1)$$

- wind kinetic energy powers the thermal energy of the shocked wind and the pressure work for the expansion

$$\dot{E}_v = \frac{1}{2}\dot{M}_v v_v^2 = \frac{dU_{int}}{dt} + p\frac{dV}{dt} = \frac{d}{dt}\left(\frac{3p}{2}\frac{4\pi\Phi}{3}R^3\right) + p\frac{d}{dt}\left(\frac{4\pi\Phi}{3}R^3\right) \qquad (2)$$

$$(1) + (2) \Rightarrow \quad R^4\dddot{R} + 12R^3\dot{R}\ddot{R} + 15R^2\dot{R}^3 = \frac{3}{2\pi\Phi}\frac{\dot{E}_v}{\rho_{HII}}$$

$$\text{solution en } R = at^n \quad \Rightarrow \quad 15n - 9 = 0 \quad \Rightarrow \quad n = 3/5$$

$$R = \left(\frac{125}{154\pi\Phi}\right)^{1/5}\left(\frac{\dot{E}_v}{\rho_{HII}}\right)^{1/5} t^{3/5}$$

SN1987a dans sa bulle

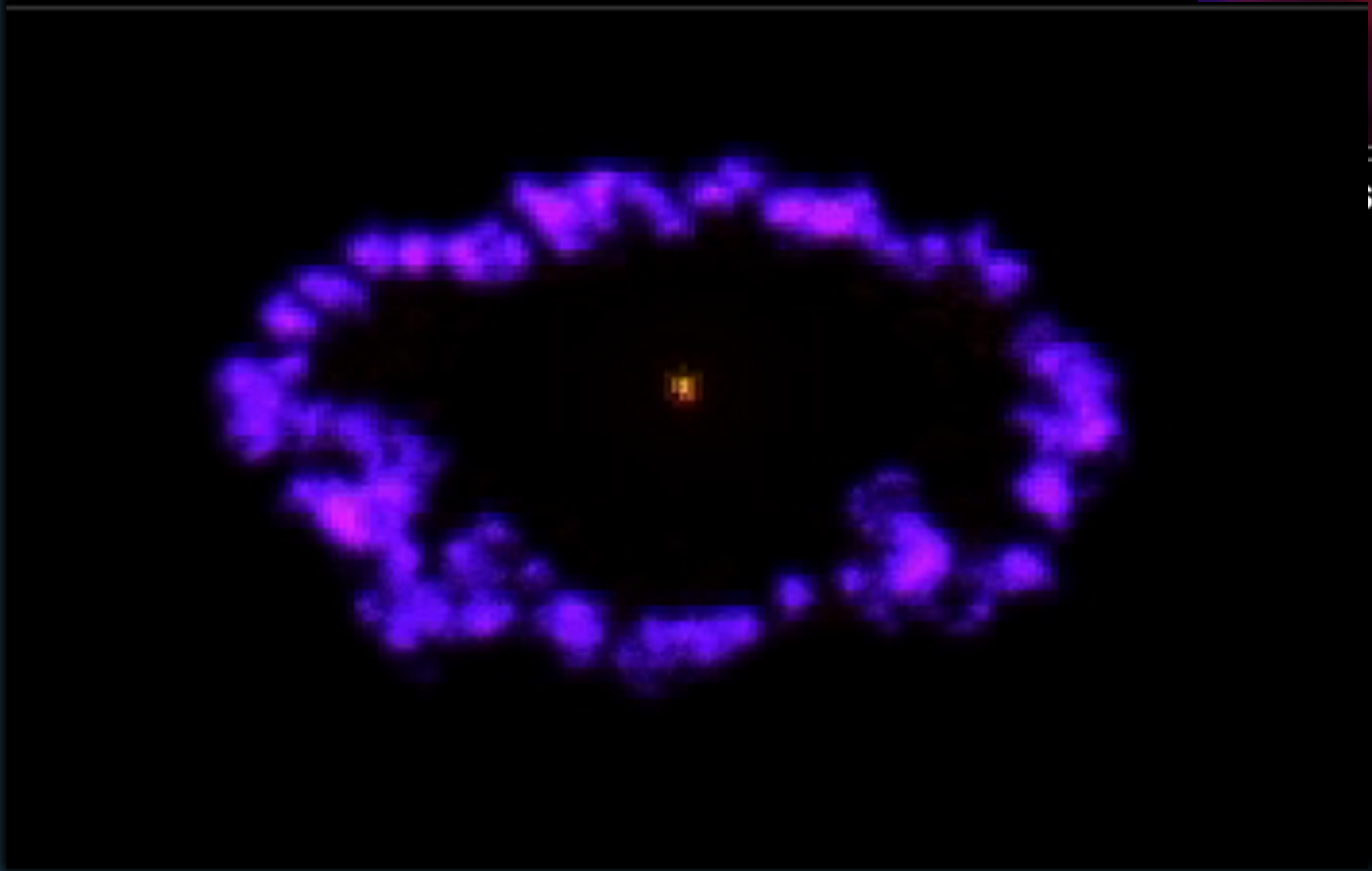
- rapid expansion in rarefied, ionized wind bubble

  e-+ions gas $\Rightarrow \langle m \rangle = 0.6\, m_p$

  shock 10^4 km/s $\Rightarrow T_2 = 1.3\ 10^9$ K …

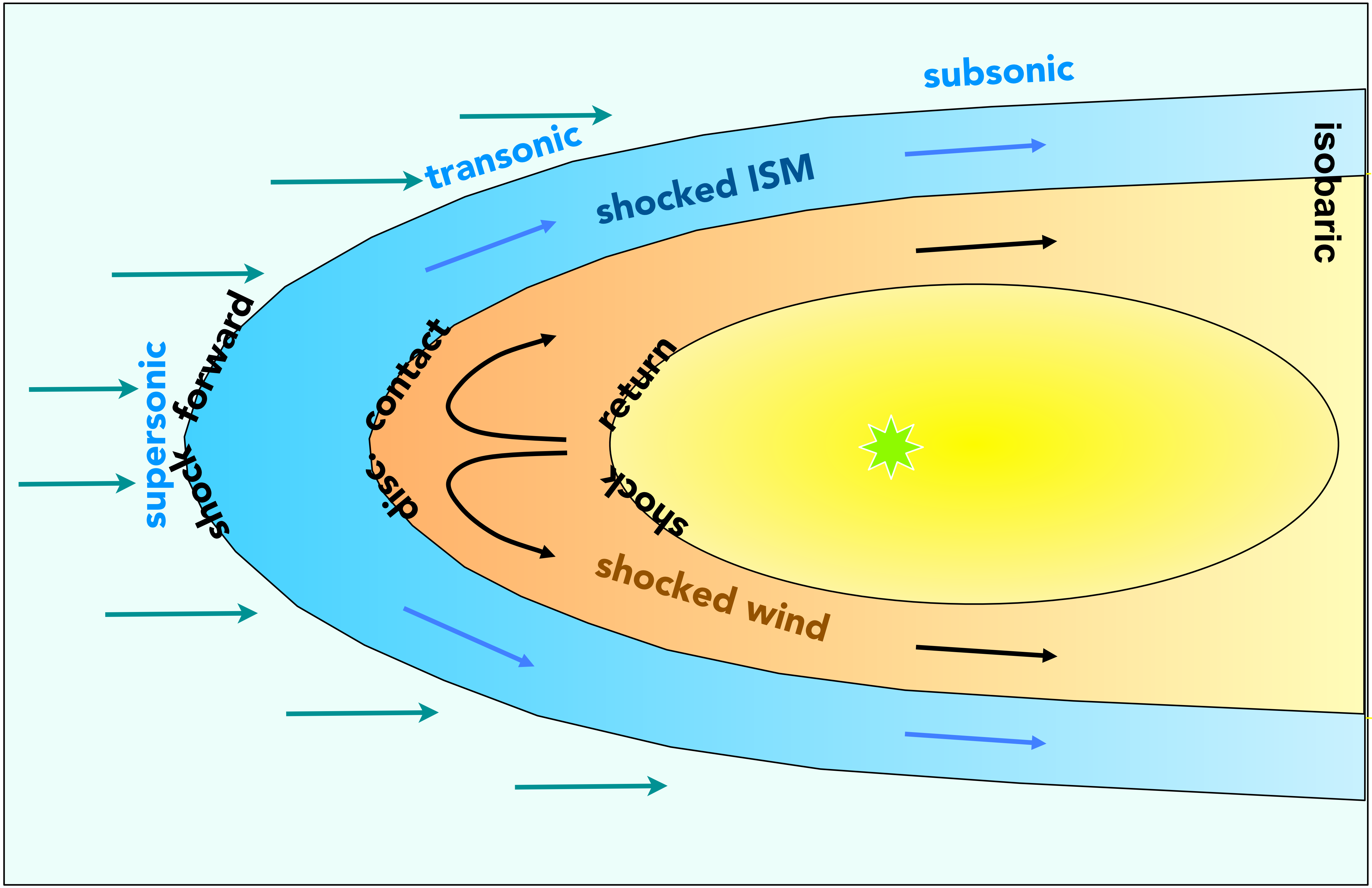
  ejecta impact on the thin bubble shell

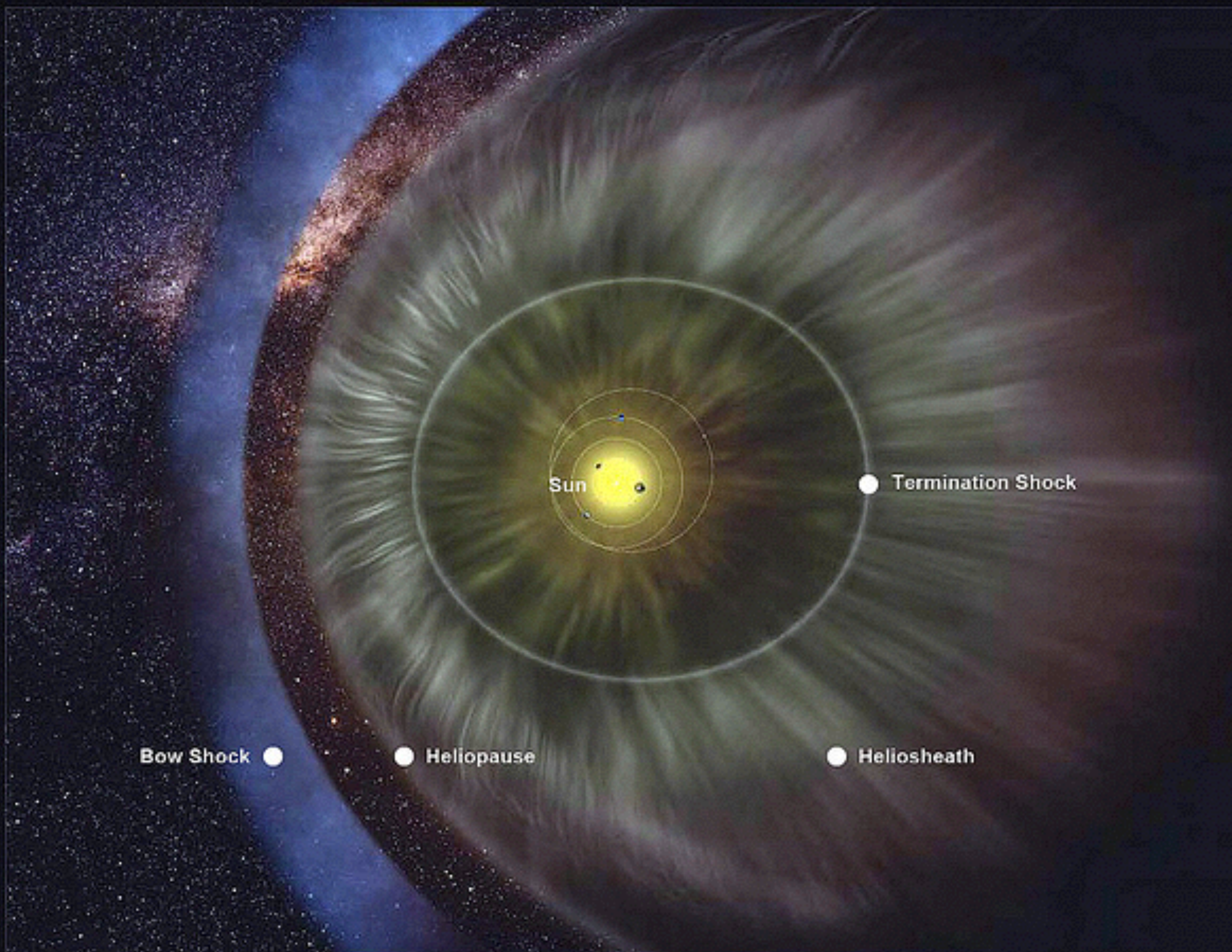
Supernova 1987A • November 28, 2003
Hubble Space Telescope • ACS



bow shocks

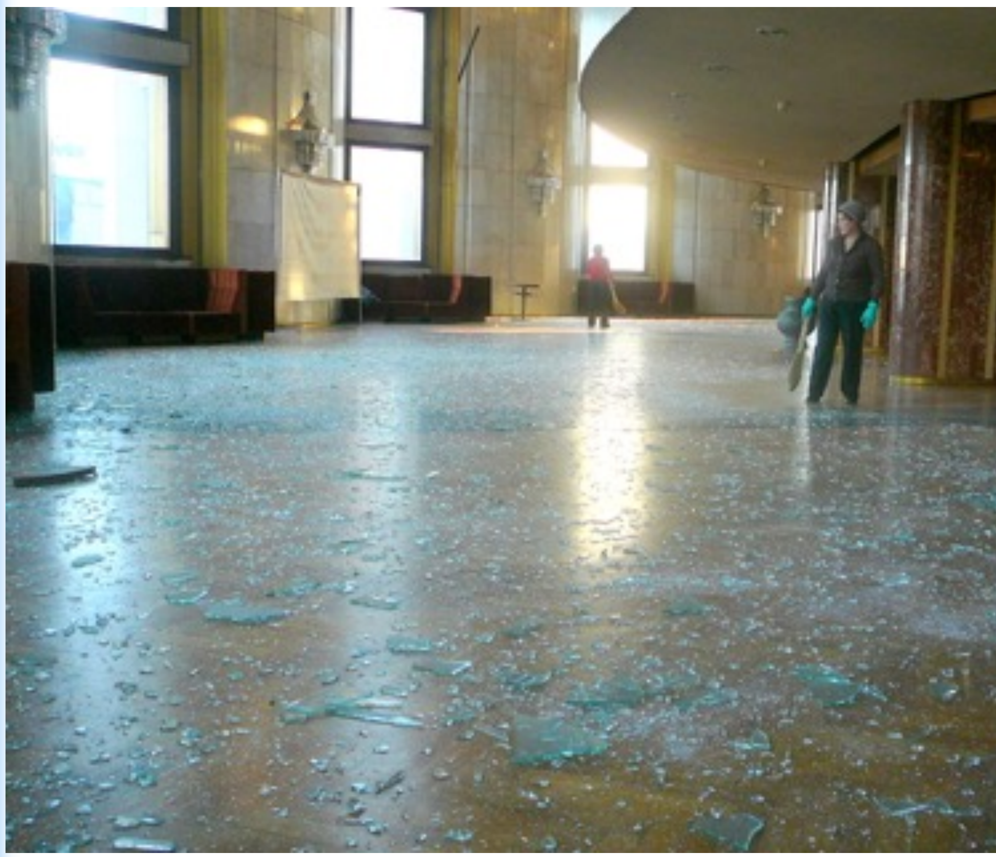
- stellar winds, pulsar winds, magnetospheric bows
- again double shock structure to slow down the outside medium and to slow down the inner wind so that they can "meet" in pressure, standing shocks

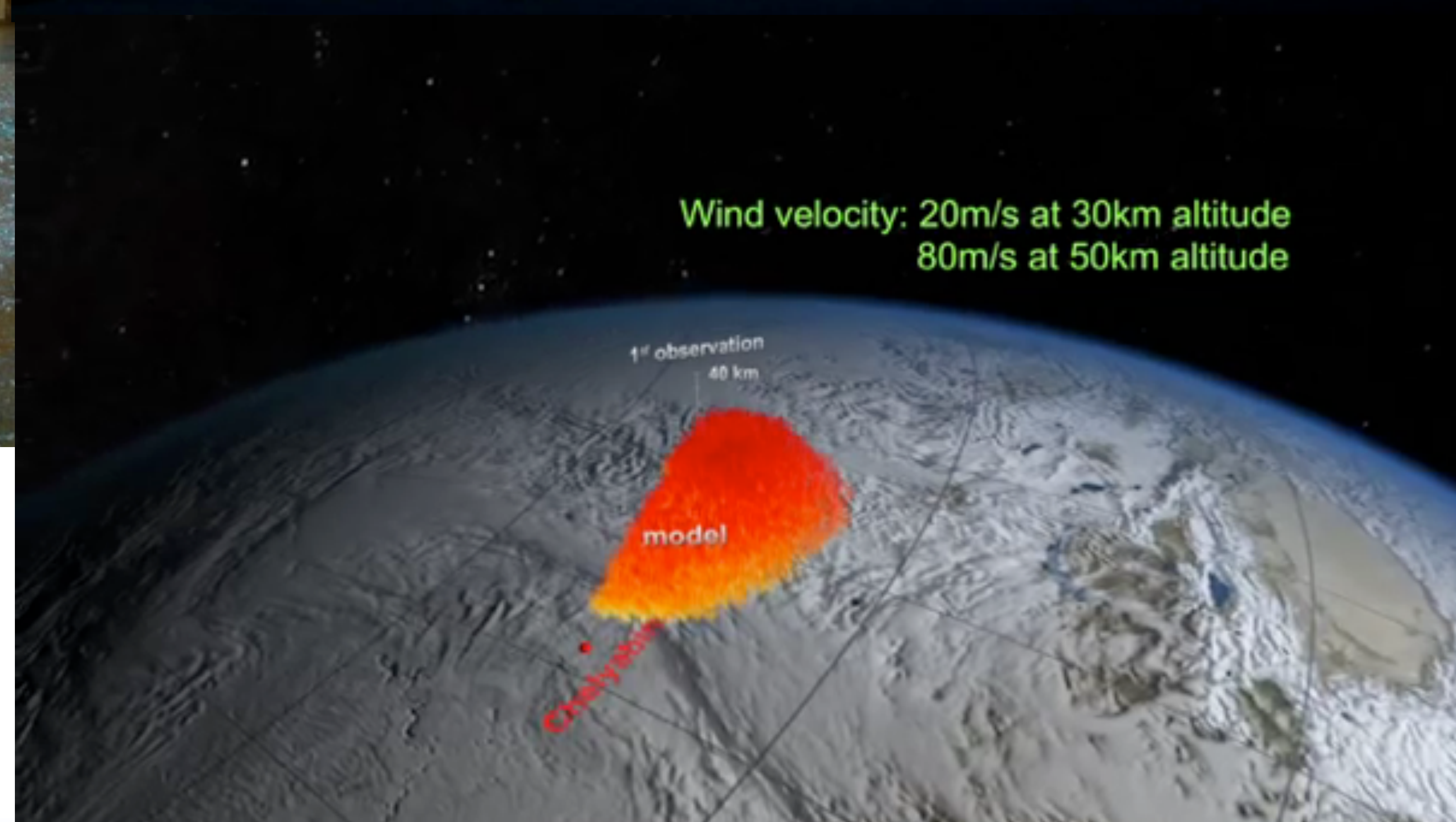



Sun
Termination Shock
Bow Shock
Heliopause
Heliosheath

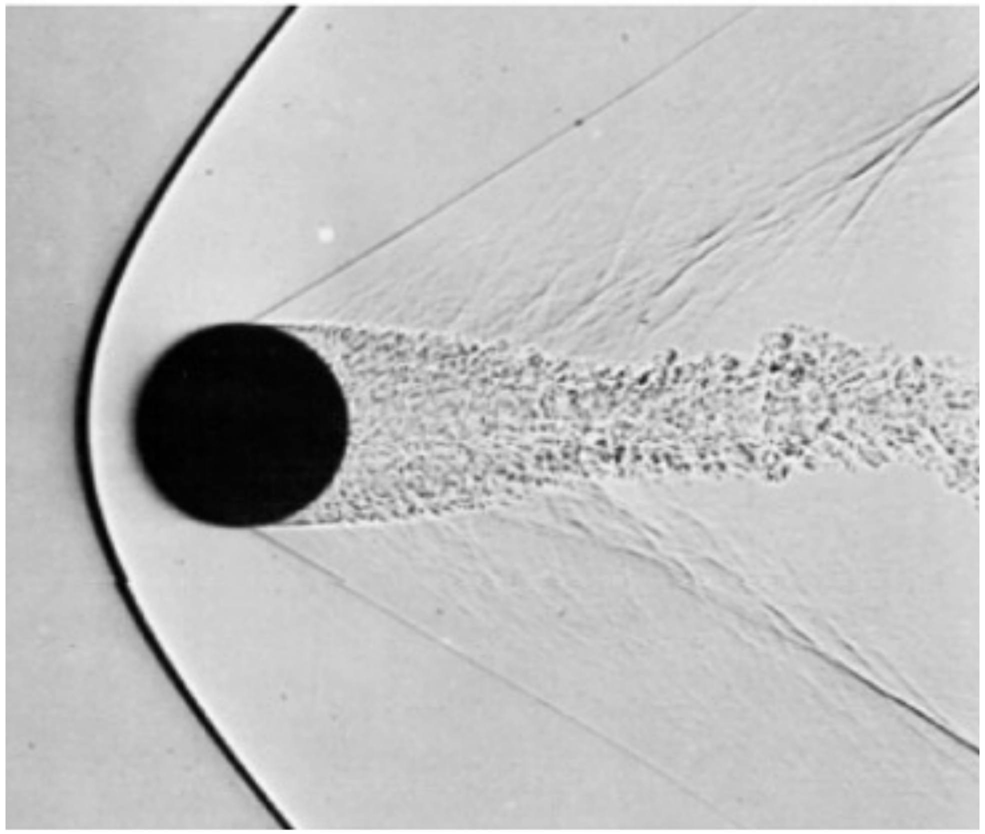
shock waves

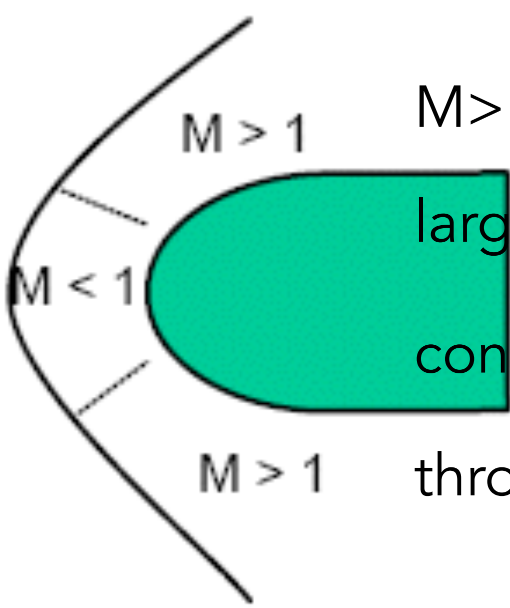
- Chelyabinsk meteor: 15/2/2013
  - ✦ 19.2 km/s => $E_{kin} \approx 1.8$ PJ
  - ✦ exploded at altitude ≈ 30 km



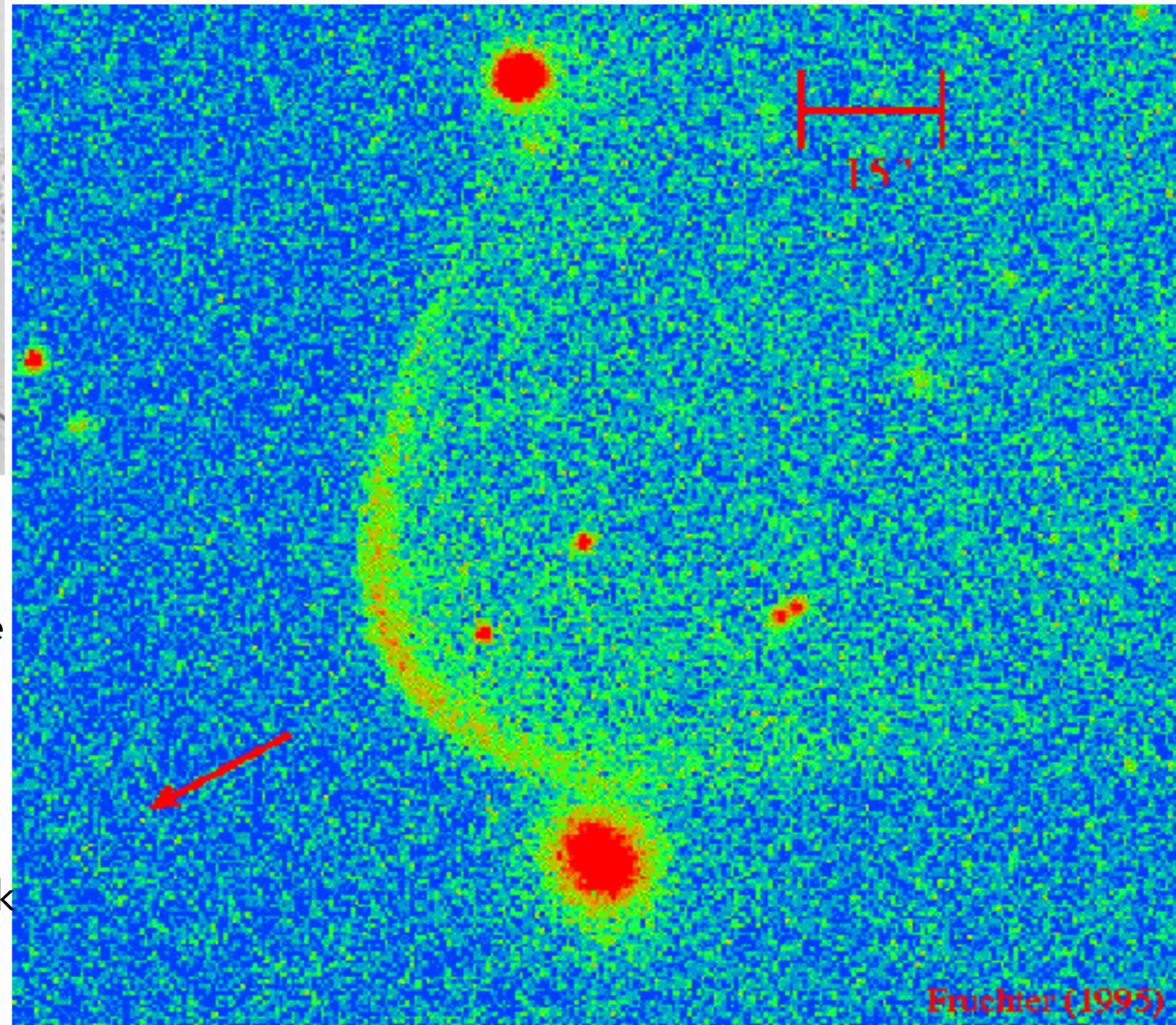


pulsar bow shock



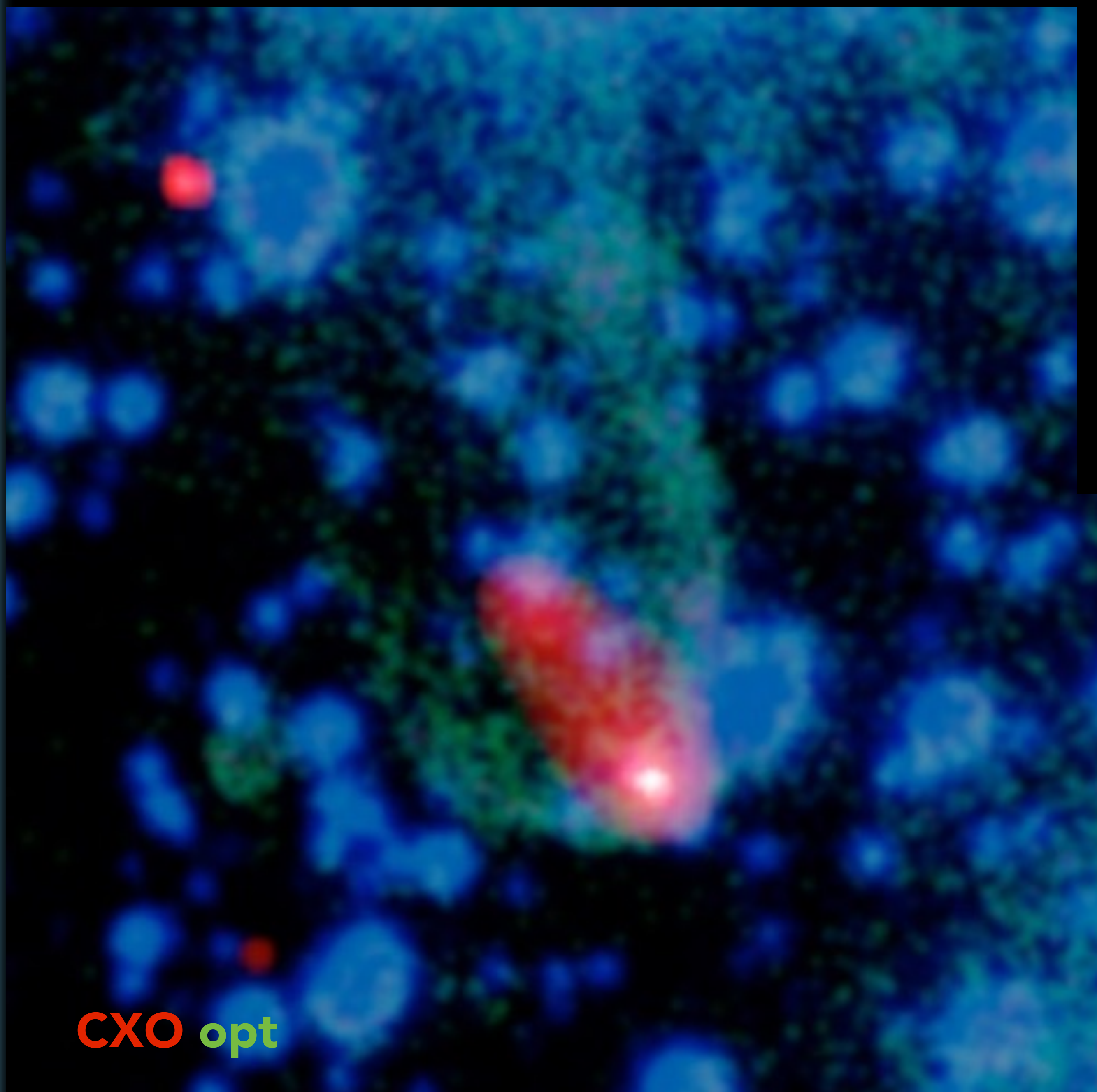


M>1 because large v// conserved through shock

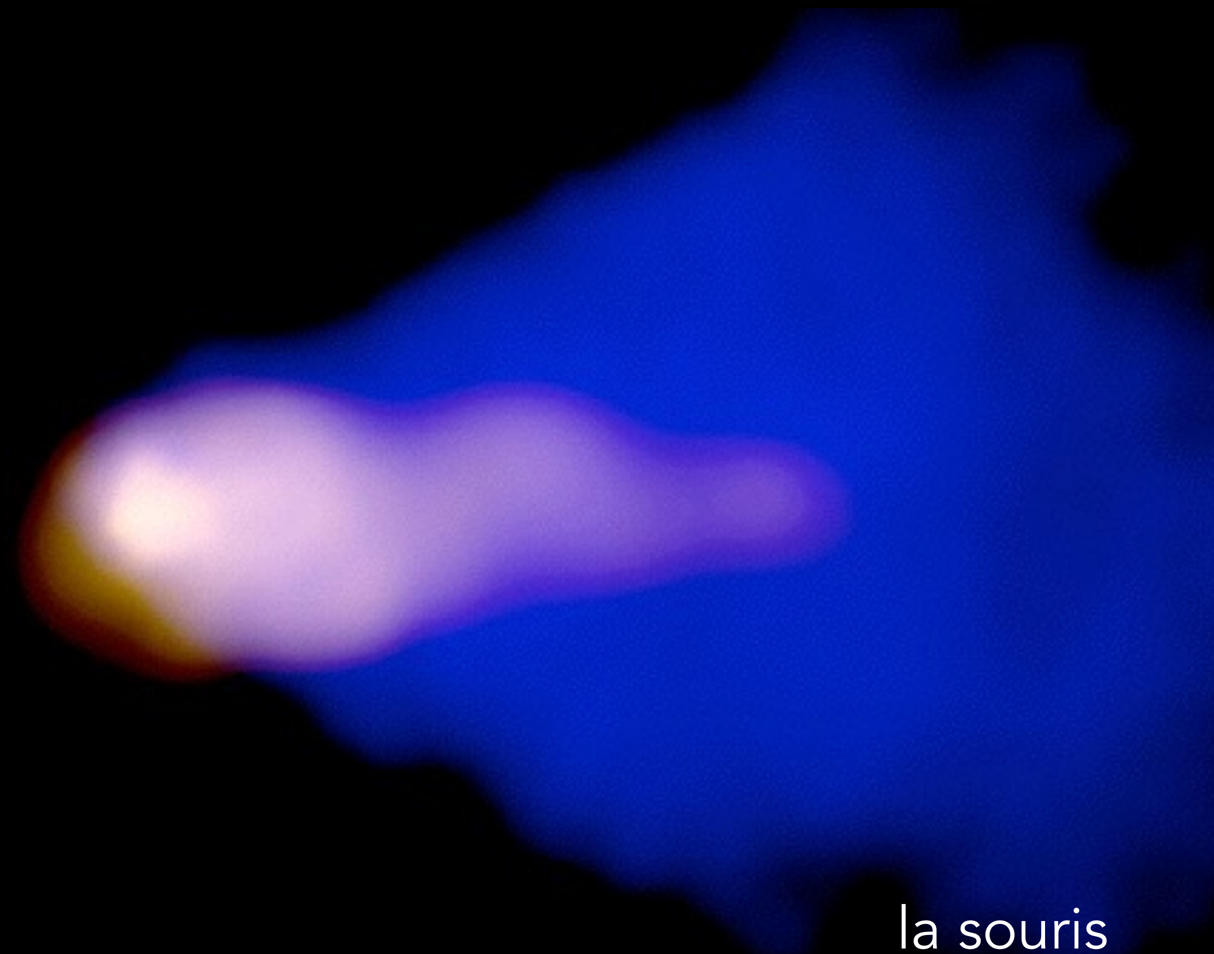


pulsar bow shock

- relativistic wind of (e+,e-) pairs (> TeV)



PSR 1957+20 veuve noire



la souris

CXO radio

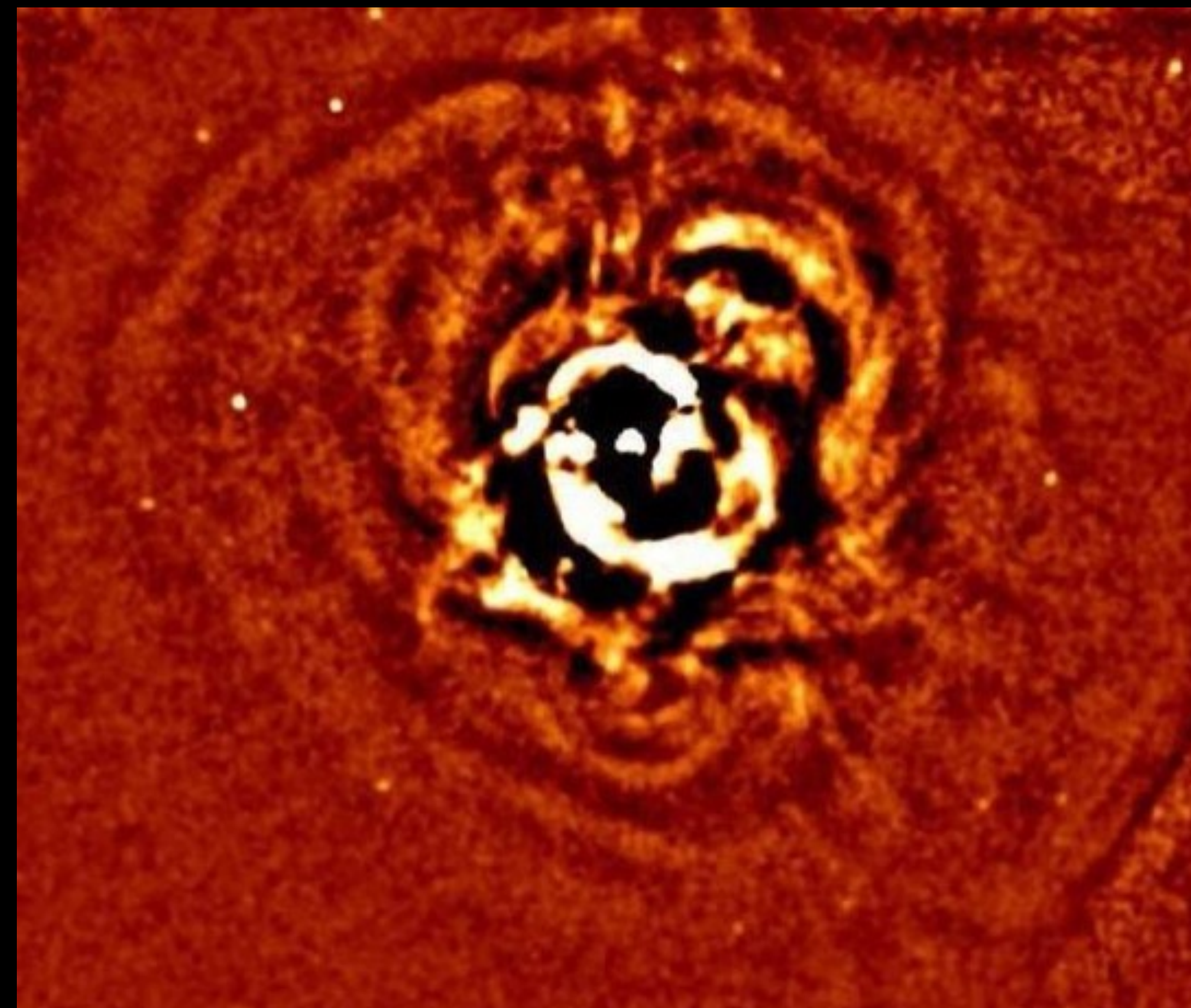
relativistic jet

- extra-galactic jet powered by the central supermassive black hole

intergalactic sculpting

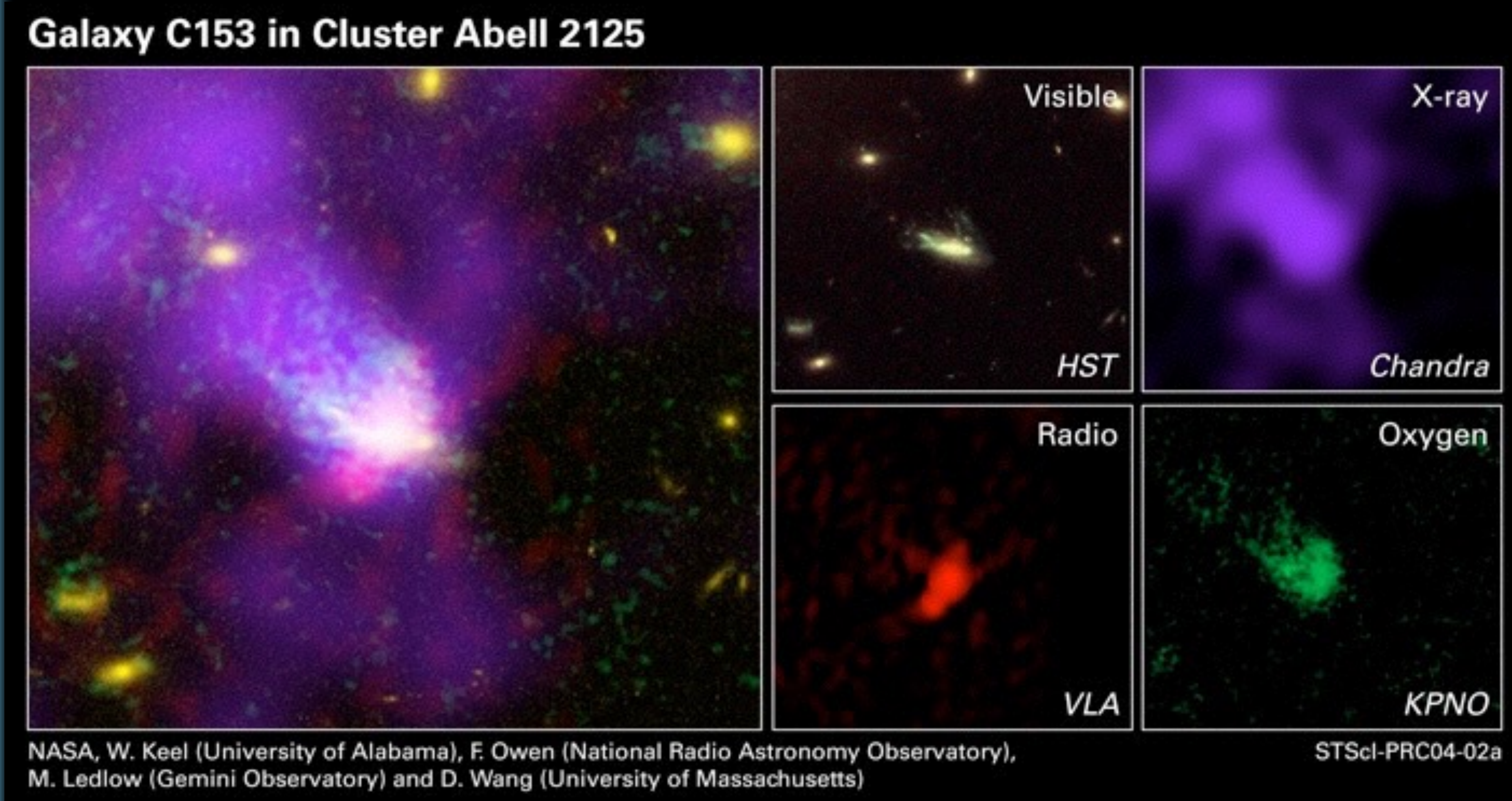
- in blue: hot gas in the cluster of galaxies (10 times more than the mass of the galaxies)

ejections



sounds

- in the large gas mass in a cluster of galaxies



shock between galaxy clusters

X
opt
masse

UnivEarthS