Influence de l'anisotropie sur les ondes hautes fréquences

Outils numériques et applications en imagerie

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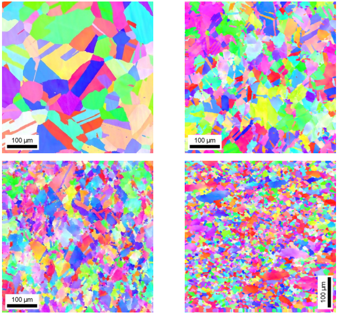
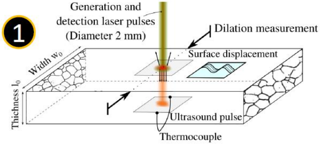
CENTRALE
PARIS

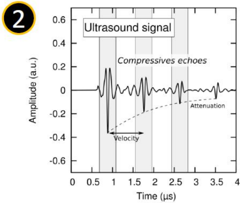
Elastic waves and anisotropy

Ultrasonic monitoring of polycrystals

- **Example**: recrystallization of inco© 718 superalloy ($NiCr_{19}Fe_{19}Nb_5Mo_3$) in hot compression.





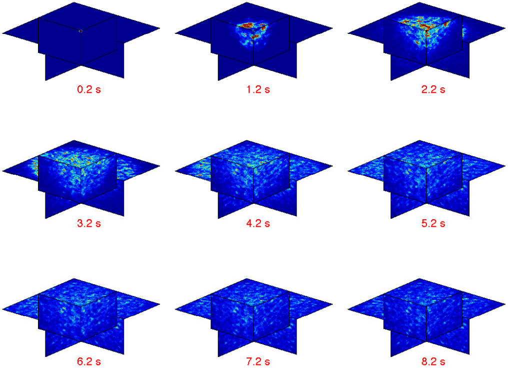


- Estimate of the grain size ℓ from the attenuation $\Sigma = \frac{2\pi}{\lambda Q}$ such that $I_{\text{out}} \equiv I_{\text{in}} e^{-\Sigma h}$:

$$
\begin{aligned}
\Sigma &\propto \ell^3 f^4 \quad && \lambda \gg \ell \text{ (Rayleigh regime)}\,, \\
\Sigma &\propto \ell f^2 \quad && \lambda \simeq \ell \text{ (stochastic regime)}\,, \\
\Sigma &\propto \ell^{-1} \quad && \lambda \ll \ell \text{ (diffusive regime)}\,.
\end{aligned}
$$

Stanke-Kino *JASA* 75(3), 665 (1984)
Goebbels *Materials Characterization for Process Control and Product Conformity*, CRC Press, Boca Raton FL (1994)

Elastic waves and anisotropy
Example: triclinic random medium



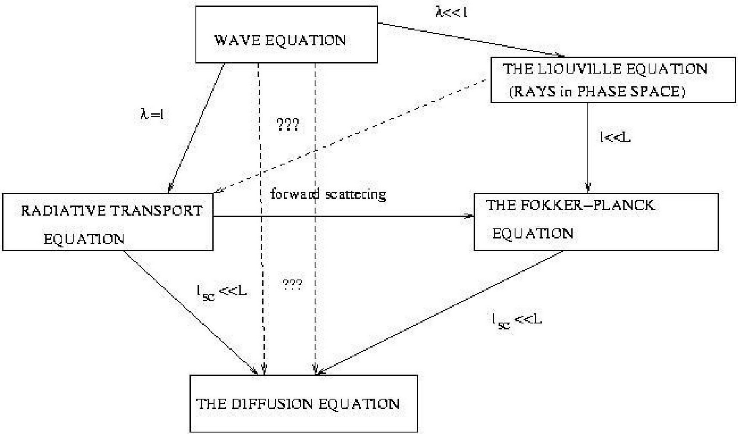
Spectral FEM with LGL + PML
$7 \cdot 10^7$ dofs, $\mathcal{O} = 5 \times 5 \times 2.5$ km^3

Velocity field in a half-space constituted by heterogeneous anisotropic materials with an homogeneous isotropic background, excited by a monopole on its surface; $\underline{c}_{\rm P} = 2000$ m/s, $\underline{c}_{\rm S} = 1000$ m/s, $\underline{\varrho} = 2000$ kg/m^3.

Ta-Clouteau-Cottereau *Eur. J. Comp. Mech.* 19(1-3), 241 (2010)

Waves in heterogeneous media

- **Scales**: λ – wave length, ℓ – correlation length, L – propagation/observation distance...



...and a **mesoscopic scale** $\ell_{\mathrm{sc}} \propto \Sigma^{-1}$ – the mean free path.

Ryzhik *HFWP'05* (2005)

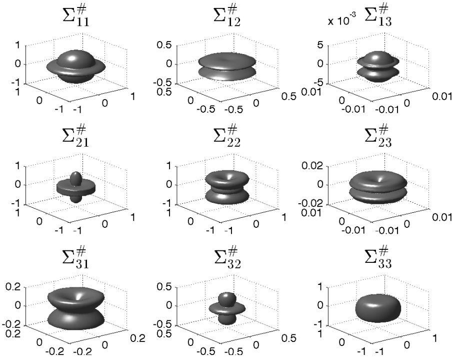
Radiative transfer in anisotropic materials

- The transport of the vibrational energy w is described by a radiative transfer model for $\lambda \simeq \ell$:

$$\{\mathcal{H}, \mathrm{w}\} = \mathcal{Q}(\mathrm{w}) \,,$$

  where $\mathcal{Q}$ is a linear collision operator accounting for elastic/inelastic scattering.

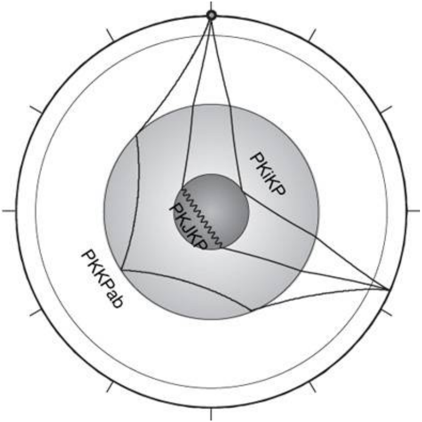
- **Example**: attenuation coefficients $\Sigma_{\alpha\beta}$ of hcp-Fe ($T = 5500°\mathrm{K}$, $P = 360\,\mathrm{GPa}$).



Vočadlo *EPSL* 254(1-2), 227 (2007)
Baydoun-Savin-Clouteau-Cottereau-Guilleminot, http://arxiv.org/abs/1402.3471

Global seismology

- **Example**: texture of the inner core from an observation of PKJKP waves.



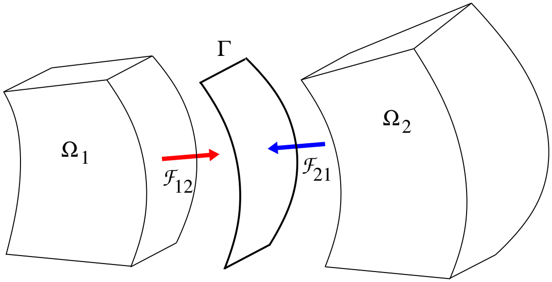
Wookey-Helffrich *Nature* 454, 873 (2008)

Quelles thématiques UnivEarthS ?

- Sciences planétaires > Texture du noyau solide ?
- Outils numériques *ad hoc* : AGHORA (Navier-Stokes 3D compressible turbulent DNS-LES-RANS en DG).

Wave propagation and flows by DG-FEM

- Consider some energy transfer between two physical domains Ω_1 and Ω_2 connected through an interface Γ:



- The evolution equations in each domain and the interface jump condition $\mathcal{F}_{12} = \mathcal{F}_{21}$ read:

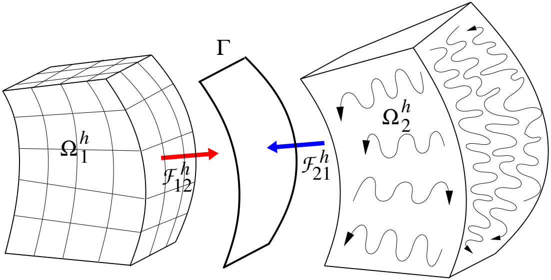
$$\{\mathcal{H}_p, \mathrm{w}_p\} = 0\,, \quad \{\mathcal{H}_p, \Gamma\}\mathrm{w}_p = \{\mathcal{H}_q, \Gamma\}\mathrm{w}_q\,, \quad p \neq q \in \{1,2\}\,.$$

- Weak form:

$$\int_{\Omega_p} \{\mathcal{H}_p, \delta\mathrm{w}\}\mathrm{w}_p = \oint_\Gamma \{\mathcal{H}_p, \Gamma\}\mathrm{w}_p \delta\mathrm{w}\,, \quad \forall \delta\mathrm{w}\,.$$

Wave propagation and flows by DG-FEM

- Consider some energy transfer between two physical domains Ω_1^h et Ω_2^h connected through an interface Γ but discretized in different ways:



- The evolution equations in each domain and the interface jump condition $\mathcal{F}_{12}^h \neq \mathcal{F}_{21}^h$ read:

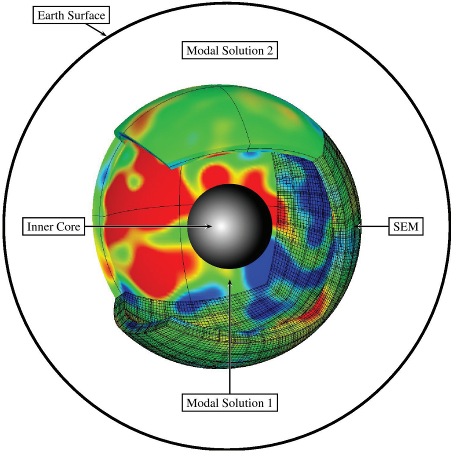
$$\{\mathcal{H}_p, \mathrm{w}_p^h\} = 0\,, \quad \{\mathcal{H}_p, \Gamma\}\mathrm{w}_p^h \neq \{\mathcal{H}_q, \Gamma\}\mathrm{w}_q^h\,, \quad p \neq q \in \{1, 2\}\,.$$

- Weak form:

$$\int_{\Omega_p^h} \{\mathcal{H}_p, \delta\mathrm{w}\}\mathrm{w}_p^h = \oint_\Gamma \mathcal{F}^*(\mathrm{w}_p^h, \mathrm{w}_q^h)\delta\mathrm{w}\,, \quad \forall\delta\mathrm{w}\,.$$

Wave propagation and flows by DG-FEM

- **Example**: coupling spectral FE – modes for the D" layer.

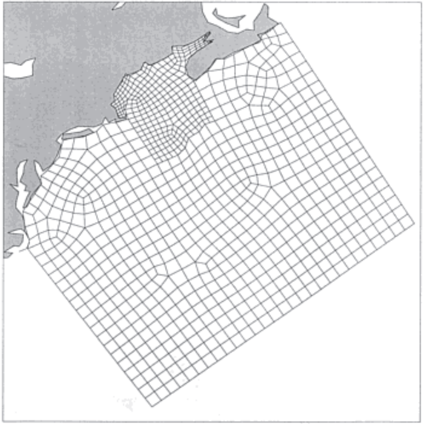


Capdeville-To-Romanowicz *GJI* 154(1), 44 (2003)

- Solids-solids, fluids-solids, Earth-atmosphere, oceanic-continental crust, MHD...

Wave propagation and flows by DG-FEM

- **Example**: non-conforming coupling for Kelvin waves in the North Atlantic.



Levin-Iskandarani-Haidvogel *Int. J. Numer. Meth. Fluids* 34(6), 495 (2000)

- Solids-solids, fluids-solids, Earth-atmosphere, oceanic-continental crust, MHD...