Cosmology and the Early Universe

Understanding Quantum Fluctuations During Inflation

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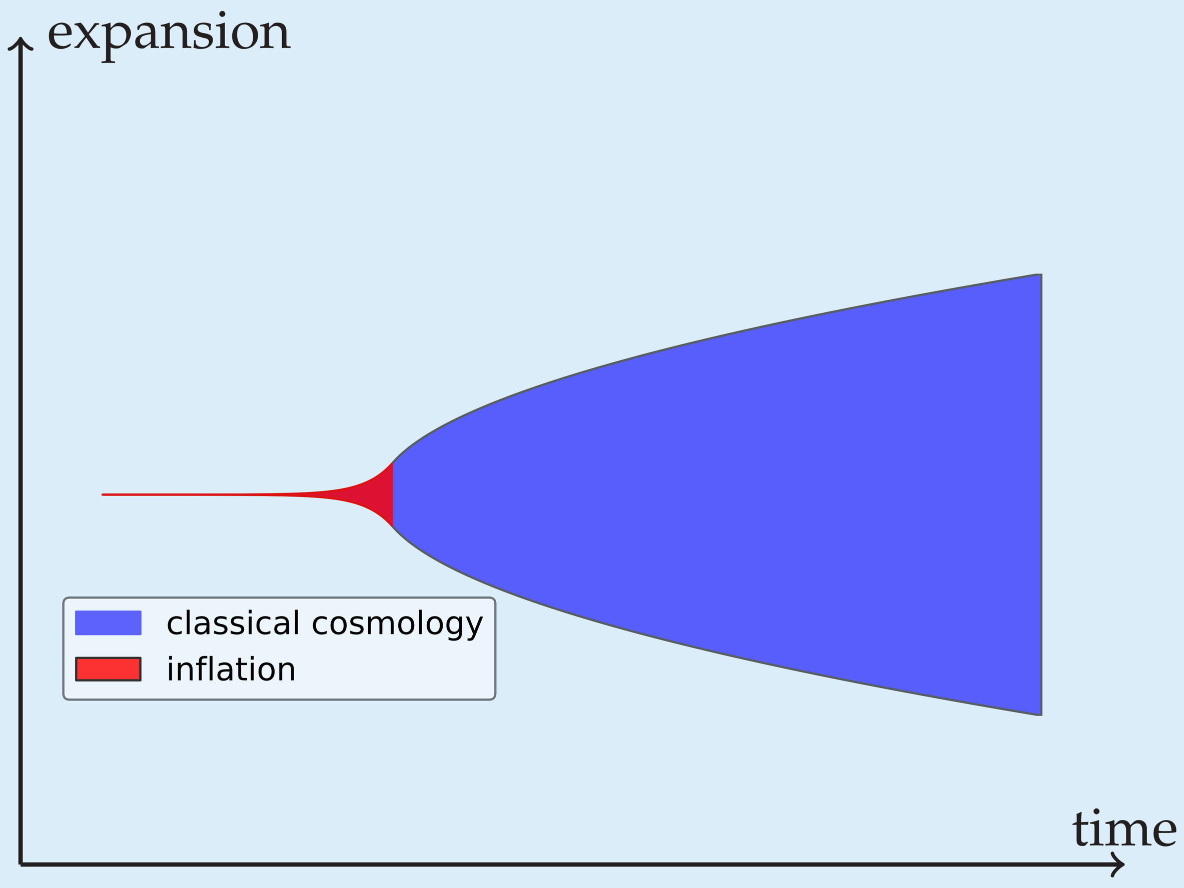
Abstract

Cosmological evidence seems to support an early phase of accelerated expansion called inflation. Models with quantum scalar fields provide a mechanism for this specific expansion, as well as for the formation of the fluctuations of density observed in the primordial universe. Although the theoretical description at first order in perturbation theory is sufficient for many practical purposes, a coherent theory of the inflationary epoch requires the understanding of specific issues of QFT in curved space. Moreover, loop corrections to perturbation theory contain divergences that signal nonperturbative properties. To overcome this difficulty, we study scalar field theory during exponential expansion by means of the nonperturbative renormalization group. This allows us to discuss properties such as the effective (renormalized) mass and coupling of the field.

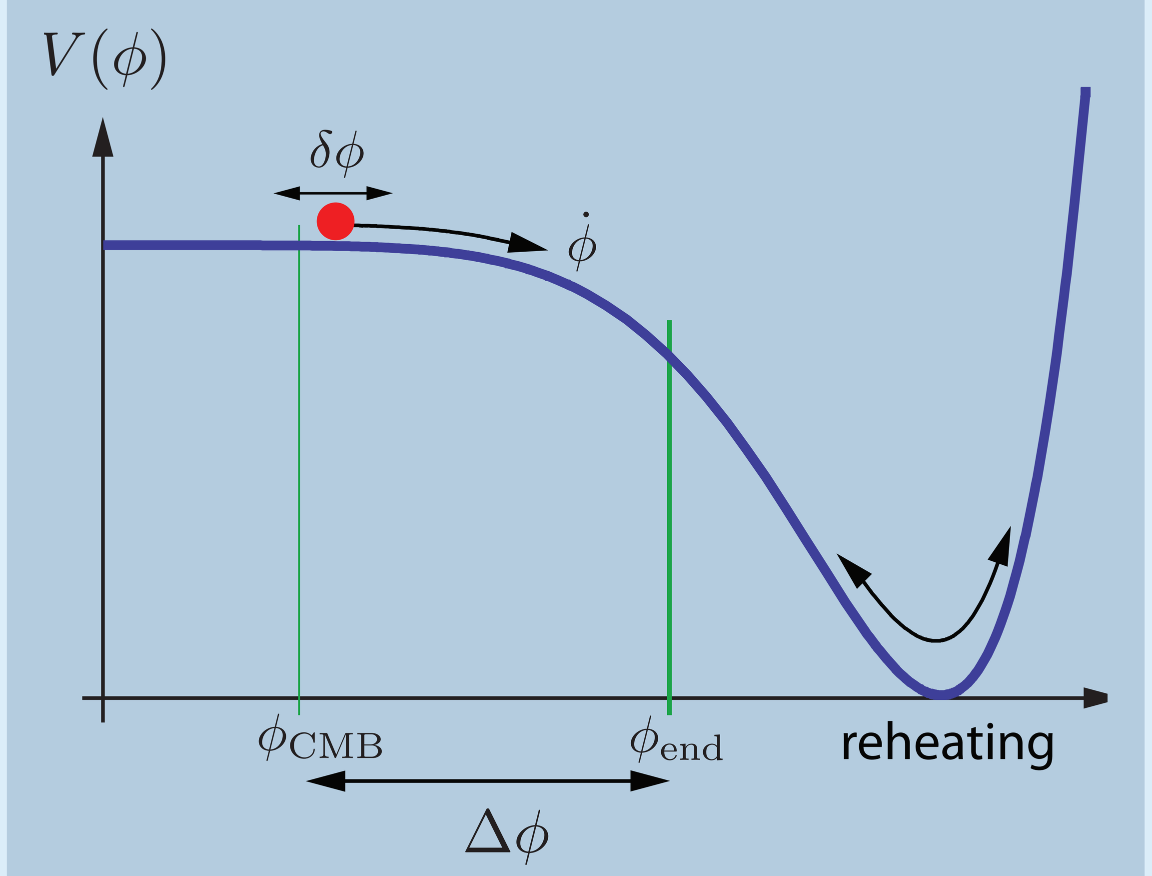
1 - Cosmology and inflation



Cosmology is the study of the history of the universe: starting from a primordial singularity, the Big Bang, some 13.8 billions years ago, it has been expanding. This rate of expansion has evolved with time, following different behaviors for different eras of cosmology depending on the content of the universe. In particular, there is evidence for a period of accelerated expansion in the first moments after the Big Bang. This period, called inflation, is especially intense: distances are increased by a factor 10^{26} in 10^{-32} seconds. This has crucial effects for the formation of structures in the universe: starting with a homogenous universe, the tiniest fluctuations in density are vastly expanded. Areas of larger density will seed the gravitational collapse of matter in stars, galaxies, etc. Therefore, our observations of the distribution of matter in the universe give us indirect information on the period of inflation.

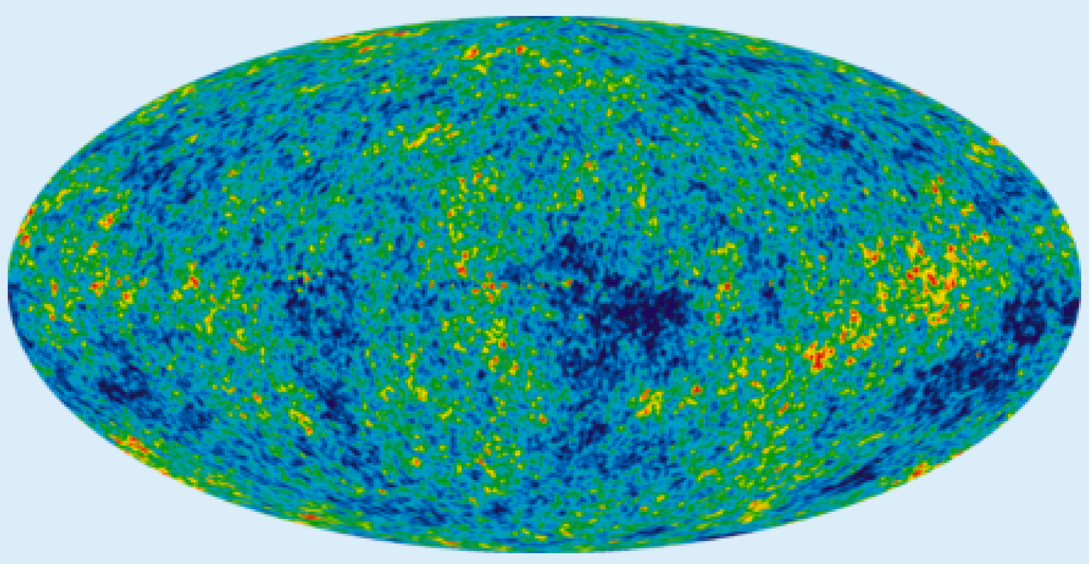
2 - Scalar fields for inflation

Although we know that a period of inflation exists in cosmology, it is unclear what it is driven by. Usual matter, such as a pressureless gas or relativistic particles, do not lead to an accelerated expansion. We are therefore led to consider more exotic models. The simplest ones involve an original content for the universe, specifically scalar fields. With the proper initial conditions and self-interacting properties, these can act as an energy source for inflation and decay to ordinary matter when it ends.



By rolling slowly down its self-interacting potential, a scalar field can source an exponential expansion of the universe for a finite period of time. Inflation ends when the scalar density reaches its minimal value. The field then decays into ordinary particles to produce the early universe as we know it.

3 - Quantum fluctuations seed large scale structures



The cosmic microwave background (CMB) is the first light emitted in the universe, 300000 years after the Big Bang. Our observations (left) tell us of an extremely homogeneous universe with small fluctuations of temperature and density. The paradigm of inflation provides an elegant explanation to these fluctuations: starting with a perfectly homogeneous scalar field, its quantum fluctuations are dramatically enhanced by the accelerated expansion so as to generate the classical fluctuations that we observe.

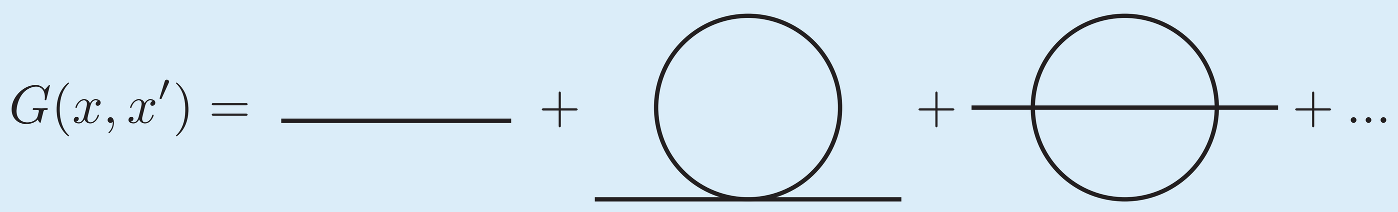
4 - Precision computation and nonperturbative aspects

The slow roll model (section 2) and the role of quantum fluctuations (section 3) have been vastly studied in the context of perturbation theory, where the full computation of the quantum behavior is discarded and only tree diagrams are computed. The full problem is much more complicated : mathematically, we must compute the functional integral

$$Z[J] = \int \mathscr{D}\varphi \, \exp\left(iS[\varphi] + i\int_x J\varphi\right)$$

Failure of the perturbative approach

The perturbative approach is a powerful tool vastly used in particle physics by means of Feynman diagrams. It is used to compute moments of the generating functional. For instance, the propagator of a field writes



where each line represents the free propagator, and loops represent integral computations. Unfortunately, in the case of scalar field during inflation, loop computations correspond to diverging integrals. This signals a nonperturbative behavior, and we must compute $Z[J]$ by other means.

Nonperturbative renormalization group

The method I have used during my PhD is the nonperturbative renormalization group (NPRG). It consists in transforming the problem into a differential equation, by introducing a renormalization scale k. The bare theory is modified by hand, so that quantum fluctuations of scale below k are effectively discarded. We then study the variation of observables with the scale k to progressively add in quantum fluctuations. By this method, we can compute quantities of interest such as correlator functions or the effective potential, that is, the renormalized mass and coupling of the theory.

Conclusion

The paradigm of inflation successfully explains our observations of the primordial universe, including the presence fo small fluctuations of density. Although many different models provide a mechanism for inflation, the simplest one remains the presence of a scalar field to source the accelerated expansion. The question of quantum fluctuations on such an expanding background is then of foremost interest, but is also highly nontrivial due to the nonperturbative nature of this problem. During my PhD, I have demonstrated that the nonperturbative renormalization group is a viable technique to tackle this issue and provide us with precision insight on inflationary physics.

References

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